| L | A | S | T | | N | A | M | E | | | | | | , | F | I | R | S | T | | N | A | M | E | |
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**Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 5**
**(Due at the start of class Wednesday, October 15, 2014. Staple this cover sheet to the front of your work.)**

| Problem: | 1 | 2 | 3 | 4 | 5 | Total | Rescaled |
|---|---|---|---|---|---|---|---|
| Your Score: | | | | | | | |
| Possible: | 20 | 20 | 20 | 20 | 20 | 100 | 10 |

**Reading:** In Chapter Three, Maps Between Spaces, read Section Three.I, Isomorphisms. The subsections are

Subsection Three.I.1 Definitions and Examples, pages 165 – 172
Subsection Three.I.2 Dimension Characterizes Isomorphism, pages 175 - 181

**Suggested Exercises:** (These 14 exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author's web site.)

Three.I.1 Exercises # 13, 15, 16, 18, 21, 22, 27, 30, 31, 35 (from pages 172 - 175)
Three.I.2 Exercises # 10, 15, 17, 20 (from pages 181 – 182)

**Assigned Exercises:** Turn in your solutions to the following five exercises, with this cover sheet stapled to the front of your work.

[1] Define map $f: \mathcal{P}_2 \to \mathbb{R}^3$ by $f(a+bx+cx^2) = \begin{pmatrix} b \\ a-b \\ b+c \end{pmatrix}$. The book would write $a+bx+cx^2 \overset{f}{\to} \begin{pmatrix} b \\ a-b \\ b+c \end{pmatrix}$.
Find the image of each of these elements of the domain: (a) $\overrightarrow{v_1} = 2-3x+4x^2$ (b) $\overrightarrow{v_2} = x+x^2$

[2] Decide whether each map $f$ is an isomorphism. If it is an isomorphism, then prove it. If it is not an isomorphism, then state a condition that it fails to satisfy.

(a) $f: \mathcal{M}_{2\times 2} \to \mathbb{R}$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a \cdot d$.

(b) $f: \mathcal{M}_{2\times 2} \to \mathbb{R}^4$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d \\ c+d \\ b+c+d \\ a+b+c+d \end{pmatrix}$.

(c) $f: \mathcal{M}_{2\times 2} \to \mathcal{P}_3$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1+(a+b)x+(b+c)x^2+(c+d)x^3$.

(d) $f: \mathcal{M}_{2\times 2} \to \mathcal{P}_3$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a+(a+b)x+(b+c)x^2+(c+d)x^3$.

[3] (a) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not an isomorphism. Why not? (Explain which of the isomorphism requirements the function fails.)
(b) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^5$ is not an isomorphism. Why not? (Explain which of the isomorphism requirements the function fails.)
(c) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ that is onto but not one-to-one. Your function must be unique, not the same function as anybody else in the class.
(d) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ that is an isomorphism. Again, your function must be unique.

[4] Consider the isomorphim $Rep_\beta: \mathcal{P}_2 \to \mathbb{R}^3$, where $\beta$ is the basis $\beta = \langle \overrightarrow{w_1}, \overrightarrow{w_2}, \overrightarrow{w_3} \rangle = \langle 1, 1+x, 1+x+x^2 \rangle$ for $\mathcal{P}_2$. Find the image of each of these elements of the domain: (a) $\overrightarrow{v_1} = 2-3x+4x^2$ (b) $\overrightarrow{v_2} = x+x^2$

[5] Every isomorphism has an inverse. For the isomorphism in problem [4], find ${Rep_\beta}^{-1}\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$