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Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 6
(Due at the start of class Monday, October 20, 2014. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: Read subsection Three.II.1.Homomorphisms: Definition (pages 183 – 188)

Suggested Exercises: (These 13 exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author's web site.)

Three.II.1 Exercises # 18, 19, 20, 21, 23, 24, 26, 30, 31, 34, 39, 41, 42 (from pages 188 - 191)

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet on front.

[1] Decide whether each map f is linear. (That is, decide if it is a homomorphism.) If it is linear, then prove it. If it is not an linear, then state a condition that it fails to satisfy.

(a) $f: \mathcal{M}_{2\times 2} \to \mathbb{R}$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a \cdot d$.

(b) $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}$. (This is an example of a "projection map".)

(c) $f: \mathcal{M}_{2\times 2} \to \mathcal{P}_3$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1 + (a+b)x + (b+c)x^2 + (c+d)x^3$.

(d) $f: \mathcal{M}_{2\times 2} \to \mathcal{P}_3$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + (a+b)x + (b+c)x^2 + (c+d)x^3$.

[2] Stating that a function is linear is different from stating that its graph is a line. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 2$ has a graph that is a line. Show that it is not a linear function

Recall that the symbol $\mathcal{F}$ is the set of real valued functions of one real variable. That is, $\mathcal{F} = \{f: \mathbb{R} \to \mathbb{R}\}$.
In the next two problems, we will deal with two subsets of $\mathcal{F}$.

- $C^0 \subset \mathcal{F}$ is the subset consisting of continuous functions.
- $C^1 \subset \mathcal{F}$ is the subset consisting of functions whose derivatives exist and are continuous.

[3] Consider the "Derivative" map That is, the map $D: C^1 \to C^0$ defined by $D(f) = \frac{df}{dx}$. Prove that D is a homomorphism.

[4] Consider the map "Definite Integral from 0 to x". That is, the map $I: C^0 \to C^1$ defined by

$$I(f) = \int_{t=0}^{t=x} f(t)dt$$

Prove that I is a homomorphism.

[5] Suppose that $f: \mathbb{R}^m \to \mathbb{R}^n$ is linear, and suppose that $f(\overrightarrow{v_1}) = \overrightarrow{w_1}, f(\overrightarrow{v_2}) = \overrightarrow{w_2}, \cdots, f(\overrightarrow{v_k}) = \overrightarrow{w_k}$ for some vectors $\overrightarrow{v_1}, \overrightarrow{v_2} \cdots, \overrightarrow{v_k}$ from $\mathbb{R}^m$ and some vectors $\overrightarrow{w_1}, \overrightarrow{w_2} \cdots, \overrightarrow{w_k}$ from $\mathbb{R}^n$.

(a) If the set $\{\overrightarrow{w_1}, \overrightarrow{w_2} \cdots, \overrightarrow{w_k}\}$ is independent, must the set $\{\overrightarrow{v_1}, \overrightarrow{v_2} \cdots, \overrightarrow{v_k}\}$ also be independent? Explain.
(b) If the set $\{\overrightarrow{v_1}, \overrightarrow{v_2} \cdots, \overrightarrow{v_k}\}$ is independent, must the set $\{\overrightarrow{w_1}, \overrightarrow{w_2} \cdots, \overrightarrow{w_k}\}$ also be independent? Explain.
(c) If the set $\{\overrightarrow{w_1}, \overrightarrow{w_2} \cdots, \overrightarrow{w_k}\}$ spans $\mathbb{R}^n$, must the set $\{\overrightarrow{v_1}, \overrightarrow{v_2} \cdots, \overrightarrow{v_k}\}$ also span $\mathbb{R}^m$? Explain.
(d) If the set $\{\overrightarrow{v_1}, \overrightarrow{v_2} \cdots, \overrightarrow{v_k}\}$ spans $\mathbb{R}^m$, must the set $\{\overrightarrow{w_1}, \overrightarrow{w_2} \cdots, \overrightarrow{w_k}\}$ also span $\mathbb{R}^n$? Explain.