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Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 8
(Due at the start of class Friday, November 7, 2014. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: In Chapter Three, Maps Between Spaces, read the following subsections:
Three.III.1: Computing Linear Maps: Representing Linear Maps with Matrices (p. 203-210)
Three.III.2: Computing Linear Maps: Any Matrix Represents a Linear Map (p. 213-218)

Suggested Exercises: Three.III.1 Exercises #12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 28 (p. 210-213)
Three.III.2 Exercises # 12, 16, 20, 21, 22, 23, 26 (p. 218-221)

Let α, β, γ be the standard bases for $\mathbb{R}, \mathcal{P}_2, \mathcal{P}_3$. That is, let $\alpha = \langle 1 \rangle$ and $\beta = \langle 1, x, x^2 \rangle$ and $\gamma = \langle 1, x, x^2, x^3 \rangle$. And let δ be the non-standard basis $\delta = \langle 1, 2x, 3x^2 \rangle$ for $\mathcal{P}_2$.

[1] Let D be the "derivative, restricted to the 3rd degree polynomials". That is, D is the map $D: \mathcal{P}_3 \to \mathcal{P}_2$ defined by $D(f) = f'$.

(a) Find $Rep_{\gamma,\beta}(D)$.

(b) Find $Rep_{\gamma,\delta}(D)$.

[2] Let $I_{[0,x]}$ be the map "definite integral from 0 to x, restricted to the 2nd degree polynomials". That is, $I_{[0,x]}$ is the map $I_{[0,x]}: \mathcal{P}_2 \to \mathcal{P}_3$ defined by

$$I_{[0,x]}(f) = \int_{t=0}^{t=x} f(t)dt$$

(a) Find $Rep_{\beta,\gamma}\left(I_{[0,x]}\right)$.

(b) Find $Rep_{\delta,\gamma}\left(I_{[0,x]}\right)$.

[3] Let $eval_5$ denote the "evaluation at 5 map, restricted to the 3rd degree polynomials". That is, $eval_5$ is the map $eval_5: \mathcal{P}_3 \to \mathbb{R}$ defined by $eval_5(f) = f(5)$. Represent the map $eval_5$ with respect to the bases γ and α.

[4] Let $h: \mathcal{P}_2 \to \mathcal{P}_3$ be a linear map such that $h(1) = x + x^2$ and $h(x) = 1 - 2x$ and $h(x^2) = 3 + x - x^2$, and let $\vec{v} = 2 - x + 3x^2 \in \mathcal{P}_2$. The goal is to find $h(\vec{v})$ using matrix operations.

(a) Find the matrix $Rep_{\beta,\gamma}(h)$.

(b) Find the column vector $Rep_{\beta}(\vec{v})$.

(c) Using the matrix-vector product, find $Rep_{\gamma}\left(h(\vec{v})\right) = Rep_{\beta,\gamma}(h) \cdot Rep_{\beta}(\vec{v})$.

(d) Using your result from (c), find $h(\vec{v})$.

[5] Suppose that $Rep_{\beta,\delta}(f) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Observe that this is the 3×3 identity matrix.

(a) What is the domain of map f? What is the codomain of f?

(b) Find $f(1 - 3x + 2x^2)$. The result should be a *function.*

(Observe that f is not the identity map, because $f(1 - 3x + 2x^2) \neq 1 - 3x + 2x^2$!)

Now let $id: \mathcal{P}_2 \to \mathcal{P}_2$ be the identity map. That is, $id(a + bx + cx^2) = a + bx + cx^2$.

(c) Find $Rep_{\beta,\beta}(id)$.

(d) Find $Rep_{\beta,\delta}(id)$. (Observe that the result is not the 3×3 identity matrix!)

Note: Part (d) originally said $Rep_{\beta,\gamma}(id)$. That was a typo!