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Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 9
(Due at the start of class Monday, November 17, 2014. Staple this cover sheet to the front of your work.)

Problem:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Your Score:											
Possible:	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**100**

Reading: In Chapter Three, Maps Between Spaces, read the following subsections:
Three.IV.1: Matrix Operations: Sums and Scalar Products (pages 222 - 224)
Three.IV.2: Matrix Operations: Matrix Multiplication (pages 226 - 231)
Three.IV.3: Matrix Operations: The Mechanics of Matrix Multiplication (pages 234 – 241)
Three.IV.4: Matrix Operations: Inverses (pages 244 – 249)
Three.V.1: Change of Basis: Changing Representations of Vectors (pages 252 – 255)
Three.V.2: Change of Basis: Changing Map Representations (pages 256 – 261)

Suggested Exercises: Three.IV.1 Exercises # 8, 12, 14, 15, 16 (pages 224 - 226)
Three.IV.2 Exercises # 14, 17, 20, 25, 26, 31, 33, 34 (pages 231 - 234)
Three.IV.3 Exercises # 24, 26, 29, 31, 32, 33, 36, 37, 41, 42 (pages 241 - 243)
Three.IV.4 Exercises # 15, 17, 18, 20, 21, 26, 27, 30 (pages 249 - 251)
Three.V.1 Exercises # 8, 11, 12, 15, 18 (pages 255 - 256)
Three.V.2 Exercises # 12, 15 (pages 262 - 263)

[1] Prove that the map $trace: \mathcal{M}_{n\times n} \to \mathbb{R}$ defined by $trace(M) = \sum_{i=1,\cdots,n} M_{i,i}$ is a linear map.

[2] Suppose that $f: U \to V$ is linear and that $g: V \to W$ is linear. Prove that $g \circ f$ is linear. Justify each step.

[3] Def.2.3 (p.227) gives a formula for the i,j entry of a matrix product: $p_{i,j} = g_{i,1}h_{1,j} + g_{i,2}h_{2,j} + \cdots + g_{i,r}h_{r,j}$ Rewrite the formula using summation notation. Be sure to indicate what values the indices i and j can have.

[4] Compute, or state "not defined". (Show details of all computations.)

(a) $\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} -3 & 1 \\ 5 & -2 \\ -2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 3 & 6 \end{pmatrix}\begin{pmatrix} -3 & 1 \\ 5 & -2 \\ -2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$

[5] Define the map $D: \mathcal{P}_3 \to \mathcal{P}_3$ by $D(f) = \frac{df}{dx}$. Let $\beta = \langle 1, x, x^2, x^3 \rangle$ be the standard basis for $\mathcal{P}_3$.

(a) Find the matrix $M = Rep_{\beta,\beta}(D)$. (b) Find the matrix $M^2 = M \cdot M$. (c) What does M^2 represent?

[6] Suppose that $A \in \mathcal{M}_{k\times m}$ and $B \in \mathcal{M}_{m\times n}$. Using summation notation, show that $(AB)^T = B^T A^T$.

[7] (a) Produce a 4×4 matrix that, when acting from the left, swaps rows 2 and 3.
(b) Produce a 3×3 matrix that, when acting from the right, swaps columns 2 and 3.

[8] Does the identity matrix represent the identity map if the bases are unequal? Explain by using an example from a previous homework assignment.

[9] Suppose that $A \in \mathcal{M}_{m\times n}$ and $B \in \mathcal{M}_{n\times m}$. Using summation notation, show that $trace(AB) = trace(BA)$.

[10] (i) Find the inverse, if it exists, by using the Gauss-Jordan Method. (ii) Check by multiplying. (iii) Also check the answers for the 2×2 matrices with Corollary 4.11 on page 249. (Show details of all computations.)

(a) $\begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 3 \\ 4 & -12 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 3 & 6 \end{pmatrix}$