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**Cover Sheet for 2014-2015 Fall Semester MATH 3200/5200 (Barsamian) Homework 10**  
**(Due at the start of class on Wednesday, December 3, 2014. Staple this cover sheet to your work.)**

<b>Problem:</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>Total</b>	<b>Rescaled</b>
<b>Your Score:</b>							
<b>Possible:</b>	<b>20</b>	<b>20</b>	<b>20</b>	<b>20</b>	<b>20</b>	<b>100</b>	<b>10</b>

**Reading:** In Chapter Five, *Similarity*, read Section II (also called *Similarity*). There are three subsections:  
 Five.II.1: Similarity: Definition and Examples (p. 386 - 387)  
 Five.II.2: Diagonalizability (p. 389 - 392)  
 Five.II.3: Eigenvalues and Eigenvectors (p. 393 - 400)

**Suggested Exercises:** Five.II.1 Exercises # 4, 6, 7, 11, 18 (p. 387 - 388)  
 Five.II.2 Exercises # 7, 8, 9, 10, 11, 14, 15, 17, 18 (p. 392 - 393)  
 Five.II.3 Exercises # 20, 21, 23, 25, 27, 28, 31, 33, 40 (p. 400 - 402)

Note: You are encouraged to check your computations using online tools, but the written work that you turn in should have complete details of all computations and clear explanations. Unjustified answers will get no credit.

- [1] Define the transformation  $t: \mathcal{P}_2 \rightarrow \mathcal{P}_2$  defined by  $t(1) = x + x^2$ ,  $t(x) = 2 - x$ , and  $t(x^2) = 1 + x - x^2$
- Find  $T = \text{Rep}_{\beta, \beta}(t)$ , where  $\beta$  is the standard basis  $\beta = \langle \vec{\beta}_1, \vec{\beta}_2, \vec{\beta}_3 \rangle = \langle 1, x, x^2 \rangle$ .
  - Find  $\hat{T} = \text{Rep}_{\gamma, \gamma}(t)$ , where  $\gamma$  is the non-standard basis  $\gamma = \langle \vec{\gamma}_1, \vec{\gamma}_2, \vec{\gamma}_3 \rangle = \langle 1, 1 + x, 1 + x + x^2 \rangle$ .
  - Find the matrix  $P$  such that  $\hat{T} = PTP^{-1}$ .
  - Check your work by computing  $PTP^{-1}$  and comparing it to your matrix  $\hat{T}$  from part (b).

- [2] Find the general formula for the positive powers of the diagonal matrix  $D = \begin{pmatrix} D_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & D_{m,m} \end{pmatrix}$ .

That is, find a formula for  $D^k$  for  $k > 0$ .

- [3] Prove that if square matrices  $A$  and  $B$  are similar, then  $A^k$  and  $B^k$  are similar for all positive integers  $k$ .
- [4] Using the method presented in the book's example 2.5 on pages 390-391, find a diagonal matrix  $D$  that is similar to the matrix  $T = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix}$ . Show all details of the calculation. (Go line-by-line through that example and adapt it to the current problem.)

- [5] Find the general formula for the positive powers of the matrix  $T = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix}$ .

That is, find a formula for  $T^k$  for  $k > 0$ .

Hint: use the results of problems [2], [3], [4].