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Cover Sheet for 2014-2015 Spring Semester MATH 3210/5210 (Barsamian) Homework 3
(Due at the start of class Monday, February 9. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: In Chapter Two, read Sections I.1, pages 77 - 86 and I.2, pages 90 - 97

Suggested Exercises: On the course syllabus and on the course web page, you will find twenty two suggested exercises from Chapter Two Sections I.1 and 1.2. These exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free online. Also note that the exams are based on the Suggested and Assigned Exercises.

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet stapled to the front of your work.

Exercises using Section Two.I.1 Concepts:

[1] Let $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 | x - 5y + 7z = 0 \right\}$. Show that S is a vector space.
(Use only Section Two.I.1 concepts. That is, prove that each of the ten vector space conditions is satisfied.)

[2] Is the set $\mathbb{Z}$ of integers a vector space over $\mathbb{R}$ under the usual addition and scalar multiplication operations?
- If you say that it is, then prove that each of the ten vector space conditions is satisfied.
- If you say that it is not, then explain which vector space conditions are not satisfied.

[3] In Example 1.12, on page 84, we were introduced to the following set, along with an operation of addition and an operation of scalar multiplication:
- Let $\mathcal{F} = \{f | f: \mathbb{R} \to \mathbb{R}\}$. That is, $\mathcal{F}$ is the set of real-valued functions of one real variable.
- For two functions f, g the symbol $f + g$ denotes the function defined by $(f + g)(x) = f(x) + g(x)$.
- For a function f and a real number r the symbol $r \cdot f$ denotes the function defined by $(r \cdot f)(x) = rf(x)$.

The author states that with those operations, the set $\mathcal{F}$ of real valued functions is a vector space, but he does not verify that any of the vector space conditions are satisfied.

Prove that the set $\mathcal{F}$, with the operations described, does qualify to be called a vector space. That is, prove that all ten of the vector space conditions are satisfied.

Exercises using Section Two.I.2 Concepts:

[4] Let V be the vector space of 2×2 matrices with real entries, with operations of matrix addition and scalar multiplication by real numbers. Let $UT = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a, b \in \mathbb{R} \right\}$. This set UT is called the *upper triangular* 2×2 matrices. The set UT is a subset of V, so it inherits the operations of addtion and scalar multiplication by real numbers. Is set UT a subspace? Explain clearly.

[5] The *odd* functions are defined as the set $Odd = \{f: \mathbb{R} \to \mathbb{R} | f(-x) = -f(x)\}$. This is a subset of the set of all real-valued functions, and so it inherits the operations of function addition and scalar multiplication by real numbers. (Those operations were defined above in problem [3].) Is the subset Odd a subspace? Explain clearly. (Your explanation will need more detail than the book's answer to a similar question.)