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Cover Sheet for 2014-2015 Spring Semester MATH 3210/5210 (Barsamian) Homework 5

(Due at the start of class Monday, March 9, 2015. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: In Chapter Three, Maps Between Spaces, read Section Three.I, Isomorphisms. The subsections are
Subsection Three.I.1 Definitions and Examples, pages 165 – 172
Subsection Three.I.2 Dimension Characterizes Isomorphism, pages 175 – 181
Also read the supplemental material "Notes on Images, Preimages, and Inverse Functions" on the web page.

Suggested Exercises: These 14 exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author's web site. Note that whenever the exercise numbering or page numbering is different for the new and old editions of the book, the information for the old edition of the book is shown in (parentheses).

Three.I.1 # 13, 17(15), 18(16), 20(18), 23(21), 24(22), 29(27), 32(30), 33(31), 37(35) on pages 172-175
Three.I.2 # 10, 15, 17, 20 from pages 179-180 (181–182)

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet stapled to the front of your work.

[1] Define map $f: \mathcal{P}_2 \to \mathbb{R}^3$ by $f(a+bx+cx^2) = \begin{pmatrix} 2c \\ b-a \\ c-b \end{pmatrix}$. The book would write $a+bx+cx^2 \xrightarrow{f} \begin{pmatrix} 2c \\ b-a \\ c-b \end{pmatrix}$.
Find the image of each of these elements of the domain: (a) $\vec{v_1} = 4 - 3x + 2x^2$ (b) $\vec{v_2} = x + x^2$

[2] Consider the isomorphim $Rep_\beta: \mathcal{P}_2 \to \mathbb{R}^3$, where β is the basis $\beta = \langle \vec{w_1}, \vec{w_2}, \vec{w_3} \rangle = \langle 1, 1+x, 1+x+x^2 \rangle$ for $\mathcal{P}_2$. Find the image of each of these elements of the domain: (a) $\vec{v_1} = 7 - 5x + 3x^2$ (b) $\vec{v_2} = x + x^2$

[3] Decide whether each map f is an isomorphism. If it is an isomorphism, then prove it. If it is not an isomorphism, then state a condition that it fails to satisfy.

(a) $f: \mathcal{M}_{2\times 2} \to \mathbb{R}$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = bc$.

(b) $f: \mathcal{M}_{2\times 2} \to \mathbb{R}^4$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a \\ b-a \\ c-b \\ d-c \end{pmatrix}$.

(c) $f: \mathcal{M}_{2\times 2} \to \mathcal{P}_3$ defined by $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1 + (a+b)x + (b+c)x^2 + (c+d)x^3$.

[4] (a) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is not an isomorphism. Why not? (Explain which of the isomorphism requirements the function fails.)
(b) The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^{(x)}$ is not an isomorphism. Why not? (Explain which of the isomorphism requirements the function fails.)
(c) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ that is onto but not one-to-one. Your function must be unique, not the same function as anybody else in the class.
(d) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ that is an isomorphism. Again, your function must be unique.

[5] Prove that if $f: \mathbb{R} \to \mathbb{R}$ is an isomorphism then f must be of the form $f(x) = kx$ where k is some non-zero real number. That is, prove that if $f: \mathbb{R} \to \mathbb{R}$ is an isomorphism then there exists a non-zero real number k such that $f(x) = kx$.