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Cover Sheet for 2014-2015 Spring Semester MATH 3210/5210 (Barsamian) Homework 6
(Due at the start of class Friday, March 13, 2015. Staple this cover sheet to the front of your work.)

Problem:	1	2	3	4	5	Total	Rescaled
Your Score:							
Possible:	20	20	20	20	20	100	10

Reading: In Chapter Three, Maps Between Spaces, read the following
Subsection Three.I.2 Dimension Characterizes Isomorphism, pages 173 – 179
Subsection Three.II.1.Homomorphisms: Definition (pages 181 – 186)
Also read the supplemental material "Notes on Images, Preimages, and Inverse Functions" on the web page.

Suggested Exercises: These 17 exercises are not to be turned in and are not graded, but you should do as many of them as possible and keep your solutions in a notebook for study. Note that detailed solutions to all of the Suggested Exercises are available in the solutions manual provided for free on the author's web site. Note that whenever the exercise numbering or page numbering is different for the new and old editions of the book, the information for the old edition of the book is shown in (parentheses).

Three.I.2 # 10, 15, 17, 20 from pages 179-180 (181–182)
Three.II.1 # 18,19,20,21,24(23),25(24),27(26),31(30),32(31),35(34),40(39), 42(41), 42(42) p186-189 (188-191)

Assigned Exercises: Turn in your solutions to the following five exercises, with this cover sheet on front.

[1] Consider the isomorphim $Rep_\beta: \mathcal{P}_2 \to \mathbb{R}^3$, where β is the basis $\beta = \langle \overrightarrow{w_1}, \overrightarrow{w_2}, \overrightarrow{w_3} \rangle = \langle 1, 1+x, 1+x+x^2 \rangle$ for $\mathcal{P}_2$. Every isomorphism has an inverse. Find ${Rep_\beta}^{-1} \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix}$.

[2] Decide whether each map f is linear. (That is, decide if it is a homomorphism.) If it is linear, then prove it. If it is not an linear, then state a condition that it fails to satisfy. Bonus: Determine if each is an isomorphism.

(a) $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix}$. (This is an example of a "projection map".)

(b) $f: \mathbb{R}^2 \to \mathcal{P}_2$ defined by $f\begin{pmatrix} a \\ b \end{pmatrix} = (a+b) + bx^2$.

(c) $f: \mathbb{R}^2 \to \mathcal{P}_1$ defined by $f\begin{pmatrix} a \\ b \end{pmatrix} = ax + b$.

[3] Stating that a function is linear is different from stating that its graph is a line. The function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 5x + 7$ has a graph that is a line. Show that it is not a linear function

Recall that the symbol $\mathcal{F}$ is the set of real valued functions of one real variable. That is, $\mathcal{F} = \{f: \mathbb{R} \to \mathbb{R}\}$.
In problems [4] and [5], we will deal with $\mathcal{F}$ and with two subsets of $\mathcal{F}$.

- $C^0 \subset \mathcal{F}$ is the subset consisting of continuous functions (continuous at every real number x).
- $C^1 \subset C^0 \subset \mathcal{F}$ is the subset consisting of functions whose 1st derivatives are continuous.

In [4] and [5] Prove that each map is a homomorphism. Bonus: Prove that each is not an isomorphism.

[4] (a) The map the "evaluation at 5 map". That is the map $eval_5: \mathcal{F} \to \mathbb{R}$ defined by $eval_5(f) = f(5)$.
(b) The map "Definite Integral from 0 to 1". That is, the map $I: C^0 \to \mathbb{R}$ defined by $I(f) = \int_{t=0}^{t=1} f(t)dt$.

[5] (a) The map "Definite Integral from 0 to x". That is, the map $I: C^0 \to C^1$ defined by $I(f) = \int_{t=0}^{t=x} f(t)dt$.
(b) The "Derivative" map That is, the map $D: C^1 \to C^0$ defined by $D(f) = \frac{df}{dx}$.