# MATH 1350 (Barsamian) Review of Limit Methods from Section 2.1 and 2.2

In Section 2.1, the limit was defined in a way that the limit could only exist if the limit was a number. Otherwise, the limit did not exist.

In Section 2.2, the definition of limit was expanded. The limit could still be a number (as in Section 2.1), but in some cases the limit could turn out to be $\infty$ or $-\infty$. (In these cases the limit would not have existed using the more restrictive definition of limit from Section 2.1.) But even in Section 2.2, it is possible for the limit to not exist. However, when the limit did not exist using Section 2.2 methods, it was usually not for the same reason that the limit did not exist using Section 2.1 methods.

| Function | $f(5)$ | Limit at $x = 5$<br>Using 2.1 Methods | Left Limit using 2.2 Methods | Right Limit using 2.2 Methods | Limit using<br>2.2 Methods |
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| $f(x) = \frac{(x-2)(x-5)}{(x-5)}$ | $f(5) = \frac{(5-2)(5-5)}{(5-5)}$<br>$= \frac{0}{0} DNE$ | $\lim_{x\to5} f(x) = \lim_{x\to5} \frac{(x-2)(x-5)}{(x-5)}$<br>$= \lim_{x\to5}(x-2)$ $(*)$<br>$= 5-2$ $(*)$<br>$= 3$<br>$(*)$ You must justify these steps! | Same method and same result as in Section 2.1.<br>That is, $\lim_{x\to5^-} f(x) = 3$ | Same method and same result as in Section 2.1.<br>That is, $\lim_{x\to5^+} f(x) = 3$ | Same method and same result as in Section 2.1.<br>That is,<br>$\lim_{x\to5} f(x) = 3$ |
| $g(x) = \frac{x-2}{x-5}$ | $g(5) = \frac{5-2}{5-5}$<br>$= \frac{3}{0} DNE$ | $\lim_{x\to5}(x-2) = 3$<br>$\lim_{x\to5}(5-2) = 0$<br>Since $\lim_{x\to5}(numerator) \neq 0$<br>and $\lim_{x\to5}(denominator) = 0$,<br>Section 2.1 Theorem 4 tells us<br>that $\lim_{x\to5} g(x) = DNE$. | When $x$ is close to 5 but slightly less than 5, the $y$-value will be<br>$y = \frac{number\ close\ to\ 3}{neg\ num\ very\ close\ to\ 0}$<br>$= very\ large\ neg\ number$<br>Conclude $\lim_{x\to5^-} g(x) = -\infty$ | When $x$ is close to 5 but slightly greater than 5, the $y$-value will be<br>$y = \frac{number\ close\ to\ 3}{pos\ num\ very\ close\ to\ 0}$<br>$= very\ large\ pos\ number$<br>Conclude $\lim_{x\to5^+} g(x) = \infty$ | Because the one-sided limits don't match, we conclude<br>$\lim_{x\to5} g(x)\ DNE$ |
| $h(x) = \frac{x-2}{(x-5)^2}$ | $h(5) = \frac{5-2}{(5-5)^2}$<br>$= \frac{3}{0} DNE$ | $\lim_{x\to5}(x-2) = 3$<br>$\lim_{x\to5}(5-2) = 0$<br>Since $\lim_{x\to5}(numerator) \neq 0$<br>and $\lim_{x\to5}(denominator) = 0$,<br>Section 2.1 Theorem 4 tells us<br>that $\lim_{x\to5} h(x) = DNE$. | When $x$ is close to 5 but slightly less than 5, the $y$-value will be<br>$y = \frac{number\ close\ to\ 3}{pos\ num\ very\ close\ to\ 0}$<br>$= very\ large\ pos\ number$<br>Conclude $\lim_{x\to5^-} h(x) = \infty$ | When $x$ is close to 5 but slightly greater than 5, the $y$-value will be<br>$y = \frac{number\ close\ to\ 3}{pos\ num\ very\ close\ to\ 0}$<br>$= very\ large\ pos\ number$<br>Conclude $\lim_{x\to5^+} g(x) = \infty$ | Because the one-sided limits match, we conclude<br>$\lim_{x\to5} g(x) = \infty$ |

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