

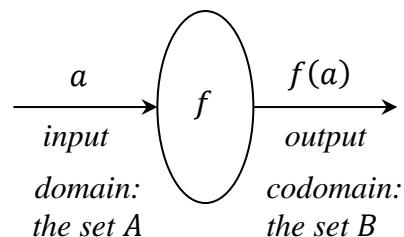
## MATH 3050 Handout: Functions

Definition: *function*

- Symbol:  $f: A \rightarrow B$
- Spoken: “ $f$  is a function from  $A$  to  $B$ ”, or “ $f$  maps  $A$  to  $B$ ”,
- Usage:  $A$  and  $B$  are sets. Set  $A$  is called the *domain* and set  $B$  is called the *codomain*.
- Meaning:  $f$  is a machine that takes as input an element of set  $A$  and produces as output an element of set  $B$ .
- Other notation: If an element  $a \in A$  is used as the input to the function  $f$ , then the symbol  $f(a)$  is used to denote the corresponding output. We say that the output  $f(a) \in B$  is the *image* of the input  $a \in A$ . If  $f(a) = b \in B$  then  $a$  is called a *preimage* of  $b$ .

The set of all possible outputs is sometimes not the whole codomain, but rather just a proper subset of the codomain. The set of all possible outputs is called the *range* of  $f$ .

In many cases, the sets  $A$  and  $B$  are sets of numbers. We will often use a *machine diagram* – a drawing like the one to the right – to visualize a function.

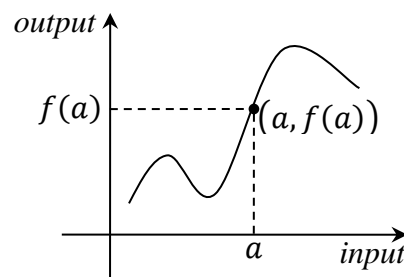


### Displaying Numerical Functions on Graphs

The *horizontal axis* (often called the “ $x$ -axis”) is used for the input numbers. For this reason, we will also refer to it as the *input axis*.

The *vertical axis* (often called the “ $y$ -axis”) is used for the output numbers. For this reason, we will also refer to it as the *output axis*.

If an input  $a \in A$  causes an output  $f(a)$  then the point  $(a, f(a))$  is on the graph, and vice-versa.



### Interpreting Graphs as Functions

You’re all familiar with functions that are described by mathematical equations, and you’re used to the idea that those functions have corresponding graphs. But now, we have a new definition of function, as a sort of “numerical machine”. Nothing in that definition requires that the function be described by an equation. In fact, sometimes a function will be described purely in words, or by a graph. If a function is going to be described only by a graph (without a corresponding equation), then we need to be sure and describe clearly how that is going to work. That is, we need to describe how the graph is going to be used as a “numerical machine”.

To interpret a graph as a function:

- Given an input number  $a$  locate the number  $a$  on the input axis (the horizontal axis) of the graph.
- Go up (or down) until you hit the graph.
- Go left (or right) until you hit the output axis (the vertical axis).
- There will be a number on the output axis at the spot where you hit. This number is to be interpreted as the output number,  $f(a)$ .

