

MATH 3050 (Barsamian) Handout: Induction

New Rule of Inference: The Principle of Induction

$P(a)$ is true
For all integers $k \geq a$, if $P(k)$ is true, then $P(k + 1)$ is true.
 \therefore For all integers $n \geq a$, $P(n)$ is true.

Usage:

- The letter “ a ” represents some fixed integer.
- The letters “ k ” and “ n ” represent variables whose domain D is the set of all integers greater than or equal to “ a ”.
- The symbol $P(n)$ represents a predicate whose domain is the set D .

This new rule of inference will be used to prove statements of the form

Statement S : “For all integers $n \geq a$, $P(n)$ is true.”

Strategy for using the principle of induction

Preliminary work:

- Identify the number playing the role of “ a ”. (Introduce it.)
- Identify the predicate $P(n)$. (Introduce it in a sentence.)
- Figure out what the expressions for $P(a)$, $P(k)$ and $P(k + 1)$ look like. (Write them down.)

Build a proof of Statement S using the following structure:

Proof of Statement S :

Basis Step: Prove that $P(a)$ is true.

(1) The proof will begin somehow.

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A bunch of steps may be involved. Usually a computation.

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(xx) $P(a)$ is true.

Inductive Step: Prove that for all integers $k \geq a$, if $P(k)$ is true, then $P(k + 1)$ is true.

(1) Suppose that k is an integer such that $k \geq a$ and that $P(k)$ is true.

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a bunch of steps may be involved

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(xx) $P(k + 1)$ is true. (some justification goes here.)

Conclusion: Therefore, for all integers $n \geq a$, $P(n)$ is true. (by the principle of induction)

End of Proof of Statement S