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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 2, Due Mon Jan 29, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1](a)(b) (Similar to suggested problems 2.3 #6, 7, 8, 12)
Two argument forms are shown at right. Use truth tables to determine whether they are valid or invalid. Be sure to indicate which columns represent the premises and which represent the conclusion, and include an explanation to support your answers.

form [1] (a)	form [1] (b)
$a \to b$	$a \to c$
$\sim a$	$b \to c$
$\therefore \sim b$	$\therefore a \vee b \to c$

[2] (Similar to suggested problems 2.3 #24, 27, 29)
An argument is shown below. The argument might be valid, or it may exhibit the converse error or the inverse error. Use symbols to write the logical form of the argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse error or the inverse error was made.

If $x^2 < 25$, then $x < 5$.
$x < 5$
Therefore, $x^2 < 25$.

[3] (Similar to suggested problem 2.3 #41) A set of premises and a conclusion are given at right. Use the valid argument forms listed in table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in example 2.3.8. Assume all variables are statement variables.

premise 1:	$a \vee b$
premise 2:	$b \to c$
premise 3:	$a \wedge d \to e$
premise 4:	$\sim c$
premise 5:	$\sim b \to f \wedge d$
conclusion:	e

[4] Let x and y be variables with domain the set **R** of real numbers.
Let $P(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$."

(a) Explain why $P(x, y)$ is false for $(x, y) = (-3,2)$.
(b) Give (x, y) values different from those in part (a) for which $P(x, y)$ is false.
(c) Explain why $P(x, y)$ is true for $(x, y) = (2,3)$.
(d) Explain why $P(x, y)$ is true for $(x, y) = (2, -3)$.
(e) Give values different from those in parts (c) and (d) for which $P(x, y)$ is true.

[5] Find the truth set of each predicate.

(a) The domain of the variable d is the set **Z**. The predicate $P(d)$ is the sentence "$6/d$ is an integer."
(b) The domain of the variable d is the set $\mathbf{Z}^+$. The predicate $P(d)$ is the sentence "$6/d$ is an integer."
(c) The domain of the variable x is the set **R**. The predicate $S(x)$ is the sentence "$1 \le x^2 \le 4$."
(d) The domain of the variable x is the set **Z**. The predicate $S(x)$ is the sentence "$1 \le x^2 \le 4$."

[6] The statement "The square of any rational number is rational" can be written formally in at least two ways:

1. "For all rational numbers x, x^2 is rational."
2. "For all real numbers x, if x is rational, then x^2 is rational."

Notice that in Version 1, the domain (the set of rational numbers) is small, and the predicate ("x^2 is rational") is just a simple statement about the numbers in the domain. And notice that in Version 2, the domain is larger, and so the predicate needs to be in the form of a conditional statement, with the hypothsis ("x is rational") serving to further narrow down the numbers being talked about. The conclusion of the second version ("x^2 is rational") is the simple statement that was the entire predicate of the first version.

The assignment continues on back. ➔

The two versions presented above could be generalized to the following two general forms:

1. For all items in some small domain, BLAH.
2. For all items in some larger domain, if the item is in the smaller subset then BLAH.

By this notation, I mean that the statement BLAH that is the predicate in the first form is the same as the statement BLAH that is the conclusion of the conditional statement in the predicate of the second form.

Rewrite each of the following statements in the two general forms described above.

(a) The derivative of any polynomial function is a polynomial function.
(b) The negative of any irrational number is irrational.
(c) The product of any two rational numbers is rational.

[7] (Based on suggested problem 3.1 #30)

Let n be a variable with domain the set **Z** of integers.
Let $Odd(n)$ be the statement "n is odd".
Let $Prime(n)$ be the statement "n is prime".
Let $Square(n)$ be the statement "n is a perfect square".
(An integer n is said to be a perfect square if it equals the square of some integer. For example, 9 is a perfect square because $9 = 3^2$.)

Consider the following two statements

Statement A: $\exists n \in \boldsymbol{Z}(Prime(n) \wedge \sim Odd(n))$".
Statement B: $\exists n \in \boldsymbol{Z}(Odd(n) \wedge Square(n))$".

Rewrite each statement without using mathematical notation (quantifiers, variables, functions, logic symbols). Determine whether the statements are true or false, and justify your answers as best as you can.

[8] (Based on suggested problem 3.1 #30) Let u, v, x, y be variables with domain the set **R** of real numbers.

Consider the following two statements

Statement A: $xy = 0 \;\Rightarrow\; x = 0 \;\vee\; y = 0$.
Statement B: $u < x \;\wedge\; v < y \;\;\Rightarrow\; uv < xy$.

Are the statements true or false? Give counterexamples for the statements that are false.

[9] (Similar to suggested problem 3.2 #5) Consider the following two statements

statement A: Some exercises have answers.
statement B: Every rational number is a real number.

Write a formal and informal negation for each statement. (Hint: start by rewriting the original statements formally, using variables and quantifiers. Then find the formal negation of the formal original, then finally find the rewrite the formal negation as an informal negation.

[10] (Similar to sugg exercises 3.2 #19, 29) Consider Statement A.

Statement A: For all integers n, if $12/n$ is an integer, then $n = 4$.

(a) Write the following four statements: $converse(A), contrapositive(A), inverse(A), \sim A$

(b) Consider the five statements $A, converse(A), contrapositve(A), inverse(A), \sim A$.
Which of those five statements is true? Explain.

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