Staple

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**2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 3, Due Fri Feb 9, 2018**

| Problem: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total | Rescaled |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Your Score: | | | | | | | | | | | | |
| Possible: | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 100 | 20 |

**[1]** (Similar to suggested problem 4.1 # 20) Show how a direct proof of Statement *A* would begin and end. That is, show the proof structure. You do not have to fill in the proof.

Statement *A*: For all real numbers $x$, if $x > 1$ then $x^2 > x$.

**[2]** (Similar to suggested problems 4.1 # 25) Consider the following statement.
Statement *B*: The sum of any two odd integers is even.
(a) Rewrite Statement *B* using variables and quantifiers.
(b) Prove or disprove Statement *B*.

**[3]** (Similar to suggested problems 4.1 # 25, 39, 41, 43) Prove or disprove Statement *C*.
Statement *C*: For all integers $n$ and $m$, if $n - m$ is even then $n^3 - m^3$ even.
Hint: To start, use long division to see if $n - m$ is a factor of $n^3 - m^3$. (This will require that you review long division of polynomials. That's why I assigned the problem.)

**[4]** (Similar to suggested problems 4.1 # 35, 53, 54) Prove or disprove Statement *D*.
Statement *D*: There exists an integer $n$ such that $6n^2 + 27$ is prime.

**[5]** (Suggested exercise 4.1 # 58) Consider the following statement.
Statement *E*: The difference of the squares of any two consecutive integers is odd.
(a) Rewrite Statement *E* using variables and quantifiers.
(b) Prove or disprove Statement *E*.

**[6]** The Zero Product Property says that if a product of two real numbers is 0, then at least one of the numbers must be 0.
(a) Write this property formally using quantifiers and variables.
(b) Write the contrapositive of your answer to (a).
(c) Write an informal version (without quantifier symbols or variables) of the contrapositive.

**[7]** (Similar to suggested exercise 4.2 # 9) Suppose that $a$ and $b$ are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $(5a + 12b)/(7a^2 b)$ must be a rational number. Hint: You will need to use [6c].

**[8]** (Suggested exercise 4.2 # 14) Consider the following statement.
Statement *F*: The square of any rational number is a rational number.
(a) Rewrite Statement *F* using variables and quantifiers.
(b) Prove or disprove Statement *F*.

**[9]** (Similar to suggested exercise 4.2 # 15) Consider the following statement.
Statement *G*: The difference of any two rational numbers is a rational number.
(a) Rewrite Statement *G* using variables and quantifiers.
(b) Prove or disprove Statement *G*.

**[10]** (Suggested exercise 4.2 # 20) Given two rational numbers $r$ and $s$, with $r < s$, prove that there is a rational number between $r$ and $s$. Hint: Use the results of 4.2 # 18, 19 (even if you were not able to do those exercises).

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