Staple

L	A	S	T		N	A	M	E							,	F	I	R	S	T		N	A	M	E	

2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 5, Due Wed Feb 28, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] (a) Let $a_k = 2k + 1$ for $k \geq 0$. Write the first five terms of the sequence. Be sure to label your terms.

(b) Let $b_k = (k-1)^3 + k + 2$ for $k \geq 0$. Write the first five terms of the sequence. Label your terms.

[2] Find an explicit formula for the sequence that begins $0, -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \frac{6}{7}, \ldots$ (Remember that an explicit formula consists of an expression for computing the terms, along with a starting value for the index.)

[3] Compute the sums and products. Your solution must contain a clear presentation of the calculation.

$$(a) \sum_{k=0}^{4} \frac{1}{3^k} \qquad (b) \prod_{k=3}^{5} (-1)^k k^3 \qquad (c) \sum_{k=1}^{998} (-1)^k \qquad (d) \prod_{k=1}^{998} (-1)^k \qquad (d) \sum_{k=1}^{1000} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

[4] Compute the following quantities. Your solution must contain a clear presentation of the calculation.

$$(a) \frac{5!}{7!} \qquad (b) \frac{5!}{0!} \qquad (c) \frac{n!}{(n-2)!} \qquad (d) \binom{7}{4} \qquad (e) \binom{7}{7} \qquad (f) \binom{n+1}{n}$$

[5] Rewrite the following using summation notation. (Do not compute the numbers!!)

$(a)\ 2 + 4 + 6 + \cdots + 738$
$(b)\ 2 + 4 + 6 + \cdots + 2n$
$(c)\ 1^3 + 2^3 + 3^3 + \cdots + 597^3$
$(d)\ 1^3 + 2^3 + 3^3 + \cdots + n^3$
$(e)\ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + 615 \cdot 616$
$(f)\ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n$

[6] Use repeated division by 2 to convert (by hand) the integer 205 from base 10 to base 2. Show the details.

[7] (Similar to book example on p.244-247, Proposition 5.2.1, and Suggested Exercise 5.2 # 1)

The goal is to use the Principle of Mathematical Induction to prove statement *S*:

Statement *S*: Any postage of at least 12 cents can be obtained by using 3 cent and 7 cent stamps.

Let *P*(*n*) be the predicate "A postage of *n* cents can be obtained using 3 cent and 7 cent stamps."

Using this definition of *P*(*n*) statement *S* can be abbreviated:

$$Statement\ S\colon\ \forall n \in \mathbf{Z}, n \geq 12(P(n))$$

Prove statement *S* using Mathematical Induction.

(Hint: Use as your model the book's proof of Prop 5.2.1 or the book's solution of Suggested Exercise 5.2 # 1.)

.

[8] The goal of this problem is to use the Method of Induction to prove statement *S*:

$$Statement\ S\text{: } \forall n \in \mathbf{Z}, n \geq 1 \left(1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n-3)}{2}\right)$$

$$Predicate\ P(n)\ is\ the\ following: \ 1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n-3)}{2}$$

Questions (a),(b),(c),(d) are similar to Suggested Exercises 5.2 # 3,4 and are about identifying the parts. This is what goes on in the portion of the *Handout on Induction* titled *"Preliminary Work"*.

(a) Write $P(1)$.

(b) Write $P(k)$.

(c) Write $P(k+1)$.

(d) In a proof by mathematical induction that the predicate $P(n)$ is true for all $n \geq 1$, what must be shown in the Inductive Step?

Question (e) is similar to Suggested Exercise 5.2 # 6, 8, 10, 13 and is about actually doing the proof. This is what goes on in the portion of the *Handout on Induction* titled *"Build a proof of Statement S using the following structure"*.

(e) Prove statement *S* using Mathematical Induction.

[9] Same questions (a),(b),(c),(d),(e) as in [8], but this time with the following Statement and Predicate.

$$Statement\ S\ is\ the\ following\text{: } \forall n \in \mathbf{Z}, n \geq 1 \left(1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2\right)$$

$$Predicate\ P(n)\ is\ the\ following\text{: } 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

[10] (Conceptual Questions)

(a) Why is 0! defined to be 1 instead of 0? Explain.

(b) In your first job after college, you are working at the *Induction Hotline*. Somebody calls wanting help with the Inductive Step. They want to know how they are supposed to prove that $P(k)$ is true if they don't know the value of k. What do you tell them?