Staple

														,												
L	A	S	T		N	A	M	E						,	F	I	R	S	T		N	A	M	E	

2017 - 2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 7, Due Fri March 30, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

[1] A *bit string* is a string of zeros and ones. A *4-bit string* is bit string with four digits. We need an organized way of listing all the 4-bit strings. The standard way is to count from 0 to 15 in binary. Fill in the table:

number in base 10	representation by a 4-bit string
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

[2] Fill in the following table by rotating your list of 4-bit strings 90° counterclockwise. The finished table shows the outputs of all possible two-place Boolean functions.

		output of f_0	output of f_1	output of f_2	output of f_3	output of f_4	output of f_5	output of f_6	output of f_7	output of f_8	output of f_9	output of f_{10}	output of f_{11}	output of f_{12}	output of f_{13}	output of f_{14}	output of f_{15}
Input	(0,0)																
	(0,1)																
	(1,0)																
	(1,1)																

[3] (a) What is $f_{11}(0,1)$?

(b) What is $f_{11}(1,0)$?

(c) For what input (a, b) is $f_4(a, b) = 1$?

(d) What is is $f_3^{-1}(1)$?

[4] The function g shown at right is a two-place Boolean function. $g(x_1, x_2) = (4x_1 + 3x_2)\ mod\ 2$

(a) What is $g(0,0)$?

(b) What is $g(0,1)$?

(c) What is $g(1,0)$?

(d) What is $g(1,1)$?

(e) Which of the sixteen two-place Boolean functions in your table from exercise [2] is g?

[5] Let $A = \{-3, -2, -1, 0\}, B = \{0, 1\}$, and let $f: \boldsymbol{R} \to \boldsymbol{R}$ be defined by the formula $f(x) = x^2$.

(a) What is $A \cap B$?

(b) What is $f(A \cap B)$?

(c) What is $f(A)$?

(d) What is $f(B)$?

(e) What is $f(A) \cap f(B)$?

(f) Prove or disprove: $\forall$ sets *A* and *B* and for all functions f, $f(A \cap B) = f(A) \cap f(B)$.

[6] Define $f: \boldsymbol{Z} \to \boldsymbol{Z}$ by $f(n) = 2n + 7$.

(a) Is f one-to-one? Prove or give a counterexample.

(b) Is f onto? Prove or give a counterexample.

(c) Is f a one-to-one correspondence? If it is, find the inverse function.

[7] Define $g: \boldsymbol{R} \to \boldsymbol{R}$ by $g(x) = 2x + 7$.

(a) Is *g* one-to-one? Prove or give a counterexample.

(b) Is g onto? Prove or give a counterexample.

(c) Is g a one-to-one correspondence? If it is, find the inverse function

[8] (a) Circle the statements that mean "f is one-to-one"

Statement *S*: $\forall x_1, x_2 (If\ x_1 = x_2\ then\ f(x_1) = f(x_2))$.

Converse of *S*: $\forall x_1, x_2 (If\ f(x_1) = f(x_2)\ then\ x_1 = x_2)$.

Inverse of *S*: $\forall x_1, x_2 (If\ x_1 \neq x_2\ then\ f(x_1) \neq f(x_2))$.

Contrapositive of *S*: $\forall x_1, x_2 (If\ f(x_1) \neq f(x_2)\ then\ x_1 \neq x_2)$.

(b) Circle the statements that mean "f is onto"

$$\forall x (\exists y (f(x) = y)).$$
$$\exists x (\forall y (f(x) = y)).$$
$$\forall y (\exists x (f(x) = y)).$$
$$\exists y (\forall x (f(x) = y)).$$

[9] $Let\ \ f(x) = \dfrac{x+a}{x+b}\ \ for\ all\ real\ numbers\ x \neq -b.$

(a) Find f^{-1}.

(b) What is the domain of f^{-1}?

[10] (a) Give an example of a function $f: \boldsymbol{R} \to \boldsymbol{R}$ that is one-to-one but not onto. **To get credit for this, your function cannot be the same as the function of anybody else in the class.**

(b) Give an example of a function $g: \boldsymbol{R} \to \boldsymbol{R}$ that is onto but not one-to-one. **To get credit for this, your function cannot be the same as the function of anybody else in the class.**

.