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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 8, Due Fri April 6, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

Remark: In [1] and [2], if you give a counterexample to demonstrate that a statement is false, your counterexample has to be unique: It can't be the same counterexample given by another student or by the book.

[1] (related to suggested exercises 7.3 # 16,18) Suppose that $f: A \to B$ and $g: B \to C$.
(a) If $g \circ f$ is one-to-one, must f be one-to-one? Prove or give a counterexample.
(b) If $g \circ f$ is one-to-one, must g be one-to-one? Prove or give a counterexample.

[2] (related to suggested exercises 7.3 # 17,19) Suppose that $f: A \to B$ and $g: B \to C$.
(a) If $g \circ f$ is onto, must f be onto? Prove or give a counterexample.
(b) If $g \circ f$ is onto, must g be onto? Prove or give a counterexample.

[3] (Suggested exercise 7.3 # 26) Prove that if if $f: A \to B$ and $g: B \to C$ are both one-to-one and onto, then $(g \circ f)^{-1}$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$
(Hint: The book gives a hint, but there is a nicer solution than the one that the book suggests. Investigate what happens when you compose the expression $g \circ f$ with the expression $f^{-1} \circ g^{-1}$, and investigate what happens when you compose $f^{-1} \circ g^{-1}$ with $g \circ f$. Then use the result of 7.3#25, which I presented in class on Friday as the "missing Theorem B".)

[4] Let $A = \{2,3,4,5\}$ and $B = \{3,4\}$. Define a *binary relation* R from A to B by $R = \{(x, y) \in A \times B | x \geq y\}$

(a) Is $2\ R\ 4$? (b) Is $4\ R\ 3$? (c) Is $(4,4) \in R$? (d) Is $(3,2) \in R$? (e) Write R as ordered pairs.

[5] (a) List all the *binary relations* from $A = \{1,2\}$ to $B = \{x, y\}$
(b) How many of the *binary relations* from part (a) are *functions*?
(c) Now suppose that C is a set with m elements and D is a set with n elements. How many *binary relations* are there from C to D?
(d) How many of the *binary relations* from part (c) are *functions*?

[6] Let $B = \{a, b, c, d\}$. Define a binary relation S on B by $S = \{(a, b), (a, c), (b, c), (d, d)\}$
Draw the directed graph of S.

[7] Let $A = \{2,3,4,5,6,7,8\}$. Define a binary relation T on A by $T = \{(x, y) \in A \times A \text{ such that } 3|(x - y)\}$
Draw the directed graph of T.

[8] Let $A = \{0,1,2,3\}$. Define a binary relation R on A by $R = \{(0,0), (0,1), (1,1), (1,2), (2,2), (2,3)\}$
(a) Draw the directed graph for R.
(b) Is R *reflexive*? (c) Is R *symmetric*? (d) Is R *transitive*? (If not, give counterexamples to demonstrate.)

[9] Let C be the unit circle relation on the set of real numbers $\boldsymbol{R}$.
That is $C = \{(x, y) \in \boldsymbol{R}^2 \text{ such that } x^2 + y^2 = 1\}$
(a) Draw a picture of the relation C showing it as a subset of $\boldsymbol{R}^2$.
(b) Is C *reflexive*? (c) Is C *symmetric*? (d) Is C *transitive*? (If not, give counterexamples to demonstrate.)

[10] Define the binary relation P on the set $\boldsymbol{Z}$ by
$$P = \{(m, n) \in \boldsymbol{Z}^2 \text{ such that } m \text{ and } n \text{ have a common prime factor}\}$$
(a) Is P *reflexive*? (b) Is P *symmetric*? (c) Is P *transitive*? (If not, give counterexamples to demonstrate.)