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2017-2018 Spring Semester MATH 3050 Section 101 (Barsamian) Homework 10, Due Wed April 25, 2018

Problem:	1	2	3	4	5	6	7	8	9	10	Total	Rescaled
Your Score:												
Possible:	10	10	10	10	10	10	10	10	10	10	100	20

On all problems, give answers in sentences, with explanations. Don't just give a number as an answer.

[1] (similar to suggested exercises 9.2#32,39)
(a) How many ways can the letters of the word *EDUCATION* be arranged in a row?
(b) How many ways can the letters of the word *EDUCATION* be arranged in a row if the letters *CAT* must remain together (in that order)?
(c) How many ways can four letters of the word *EDUCATION* be selected and arranged in a row?
(d) How many ways can four letters of the word *EDUCATION* be selected and arranged in a row if the first two letters must be *DT* (in that order)?

[2] (similar to suggested exercises 9.2#41,42) Prove that $\forall n \geq 4\big(P(n+1,4) - P(n,4) = 4P(n,3)\big)$.

[3] (sim to Example 9.3.3 and sugg ex 9.3# 3,16) Your credit card needs a PIN, which can be any four digits.
(a) How many PINs are possible?
(b) How many PINs do not have any repeated digits? ("Repeated" means digits used more than once. It does not mean that the repeated digits have to be adjacent in the PIN.)
(c) How many PINs do have repeated digits?
(d) What is the probability that a randomly chosen 4-digit PIN does have repeated digits?

[4] (similar to Examples 9.1.4 and 9.3.6 and suggested exercise 9.3#23)
(a) How many integers from 1 through 1000 are multiples of 5 or multiples of 7?
(b) Suppose that an integer from 1 through 1000 is chosen at random. What is the probability that the integer is a multiple of 5 or a multiple of 7?
(c) How many integers from 1 through 1000 are neither multiples of 5 nor multiples of 7?

[5] (similar to suggested exercise 9.3#31) **The Birthday Problem Part 1: Group of 5 people**
Assume that all years have 365 days, and all birthdays have equal probability.
Given a group of five people, *A,B,C,D,E*,
(a) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur?
(b) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur so that no two people share the same birthday?
(c) What is the total number of ways in which the birthdays of *A,B,C,D,E* could occur so that at least two people share a birthday?
(d) What is the probability that at least two people in the group *A,B,C,D,E* share a birthday?

[6] (similar to suggested exercise 9.3#32) **The Birthday Problem Part 2: Group of k people**
We are interested in the probability that at least two people in a group of k people share a birthday. We will define $P(k)$ to be the probability that at least two people in a group of k people share a birthday.
Our goal is to find a formula for $P(k)$, make a table of values and a graph for it, and use that table of values to answer some questions. But before doing that, we can observe the following:

- In problem [5](d), you found the value of $P(5)$.
- If $k = 1$, then there is only one person in the group, so there is no way that two people in the group can share a birthday. That is, $P(1) = 0$.
- If $k = 366$ or more, then at least two people must share a birthday, because there are more people then birthdays. Since a shared birthday is certain, we can say that $If\ k \geq 366\ \ then\ P(k) = 1$.
- In general, as k increases in the interval $1 \leq k \leq 366$, the value of $P(k)$ should increase from 0 to 1.

We will now work towards getting a formula for $P(k)$.

(a) What is the total number of ways in which the birthdays of the k people in the group could occur?
(b) What is the total number of ways in which the birthdays of the k people in the group could so that no two people share the same birthday?
(c) What is the total number of ways in which the birthdays of the k people in the group could occur so that at least two people share a birthday?
(d) What is the probability that at least two people in the group share a birthday? That is, what is $P(k)$?
(e) Using Desmos, make a table of values for $P(k)$, for $n = 1,2, \ldots, 50$, along with a graph of that data. (Integer domain, not real numbers: the graph should be dots.) Make your graph large and clear, so that it the curve spans the whole screen and the axes and their labels are clearly visible. Print your graph.
(f) Using your data, how many large does k need to be for $P(k)$ to be at least 0.5? Illustrate on your graph.

[7] (similar to suggested problem 9.5#5) Use Theorem 9.5.1 to compute each of the following.
(Show how the expressions can be simplified. Do not use a calculator!)

(a) $\binom{7}{0}$ (b) $\binom{7}{1}$ (c) $\binom{7}{2}$ (d) $\binom{7}{3}$ (e) $\binom{7}{4}$ (f) $\binom{7}{5}$ (g) $\binom{7}{6}$ (h) $\binom{7}{7}$

[8] (similar to Examples 9.5#4,5,6,7 and sugg ex 9.5#5) A computer programming department has 15 members.
(a) How many ways can a group of six be chosen to work on a project?
(b) Suppose eight department members are women and seven are men.
(i) How many groups of six can be chosen that contain four women and two men?
(ii) How many groups of six can be chosen that contain at least one man?
(iii) How many groups of six can be chosen that contain at most three women?
(c) Suppose two department members refuse to work together on projects. How many groups of six can be chosen to work on a project?
(d) Suppose two department members insist on either working together or not at all on projects. How many groups of six can be chosen to work on a project?

[9] (similar to suggested exercise 9.5# 17) (similar to suggested problem 9.5#17) Eight points labeled *A,B,C,D,E,F,G,H* are arranged in a plane in a way that no three of them lie on the same line.
(a) How many straight lines are determined by the eight points?
(b) How many of those straight lines do not pass through point A?
(c) How many triangles have three of the eight points as vertices?
(d) How many of these triangles do not have A as a vertex?

[10] (similar to suggested problem 9.5#16) Suppose that three computer boards in a production run of fifty are defective. A sample of five is to be selected to be checked for defects.
(a) How many different samples can be chosen?
(b) How many samples will contain at least one defective board?
(c) What is the probability that a randomly chosen sample of five contains at least one defective board?

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