Class Drill for Section 2.2: Justifying and Illustrating a Proof of Fano's Theorem #3

Axiom System:	Fano's Geometry
Primitive Objects:	*point*, *line*
Primitive Relation:	*The point lies on the line.*
Axioms:	<F1> There exists at least one *line*. <F2> For every *line*, there exist exactly three *points* that *lie on* the *line*. <F3> For every *line*, there exists a *point* that does not *lie on* the *line*. <F4> For any two *points*, there is exactly one *line* that both *points lie on*. <F5> For any two *lines*, there exists a *point* that *lies on* both *lines*.

Fano's Geometry Theorem #3: There exist exactly seven points.

Justify the steps in the following Proof of Fano's Theorem #3

Part 1: Show that there must be at least seven points.

Introduce Line L_1 and points *A, B, C, D*.

(1) There exists a line. **(justification: _____)**
We can call it L_1. **(Make a drawing.)**

(2) There are exactly three points on L_1. **(justification: _____)**
We can call them *A*, *B*, *C*. **(Make a new drawing.)**

(3) There must be a point that does not lie on L_1. **(justification: _____)**
We can call it *D*. **(Make a new drawing.)**

Introduce Line L_2 and point E.

(4) There must be a line that both A and D lie on. **(justification: _____)**

(5) The line that both A and D lie on cannot be L_1. **(justification: _____**

_____)

So it must be a new line. We can call it L_2. **(Make a new drawing.)**

(6) There must be a third point that lies on L_2. **(justification: _____)**

(7) The third point on L_2 cannot be B or C. **(justification: _____**

_____

_____)

So it must be a new point. We can call it E. **(Make a new drawing.)**

Introduce Line L_3 and point F.

(8) There must be a line that both B and D lie on. **(justification: _____)**

(9) The line that both B and D lie on cannot be L_1 or L_2. **(justification: _____**

_____

_____)

So it must be a new line. We can call it L_3. **(Make a new drawing.)**

(10) There must be a third point that lies on L_3. **(justification: _____)**

(11) The third point on L_3 cannot be A, C, or E. **(justification: _____**

_____

_____

_____)

So it must be a new point. We can call it F. **(Make a new drawing.)**

Introduce Line L_4 and point G.

(12) There must be a line that both C and D lie on. **(justification: _____)**

(13) The line that both C and D lie on cannot be L_1 or L_2 or L_3. **(justification: _____**
_____
_____
_____)

So it must be a new line. We can call it L_4. **(Make a new drawing.)**

(14) There must be a third point that lies on L_4. **(justification: _____)**

(15) The third point on L_4 cannot be A, B, E, or F. **(justification: _____**
_____
_____
_____
_____)

So it must be a new point. We can call it G. **(Make a new drawing.)**

We have shown that there must be at least seven points, so this concludes Part 1.

Part 2: Show that there cannot be an eight point.

(16) Suppose there is an eighth point. **(justification: _____ _____)** Call it H.

(17) There must be a line that both A and H lie on. **(justification: _____)**

(18) The line that both A and H lie on cannot be L_1 or L_2 or L_3 or L_4. **(justification: _____ _____ _____ _____)**

So it must be a new line. We can call it L_5.

(19) There must be a third point that lies on L_5. **(justification: _____)**

(20) Line L_5 must intersect each of the lines L_1 and L_2 and L_3 and L_4. **(justification: _____ _____)**

(21) The third point on L_5 must be D. **(justification (Be sure to explain clearly): _____ _____ _____ _____ _____)**

(22) So points A, D, H lie on L_5.

(23) We have reached a contradiction. **(explain the contradiction: _____ _____ _____ _____)**

Therefore, our assumption in step (16) was wrong. There cannot be an eighth point.

End of proof