Class Drill 2: Drill for Section 2.3: Two Theorems of Incidence Geometry

Axiom System:	Incidence Geometry (Introduced in Section 2.3.2)
Primitive Objects:	*point*, *line*
Primitive Relations:	*the point lies on the line*
Axioms:	<I1> There exist two distinct points. <I2> For every pair of distinct points, there exists exactly one line that both points lie on. <I3> For every line, there exists a point that does not lie on the line. <I4> For every line, there exist two points that do lie on the line.

In this class drill, you will study the proofs of some of the Incidence Geometry theorems. There are two parts to the class drill. They are labeled [1], [2]

[1] Justify the steps in the following proof of Incidence Geometry Theorem 2.

Incidence Geometry Theorem #2: In Incidence Geometry, there exist three points that are not collinear.

Proof

(1) In Incidence Geometry, two distinct points exist. We can call them *P* and *Q*. (Justify.) (Make a drawing)

(2) A line exists that passes through *P* and *Q*. We can call it *L*. (Justify.) (Make a new drawing)

(3) There exists a point that does not lie on *L*. We can call it *R*. (Justify.) (Make a new drawing)

(4) We already know (by statement (3)) line *L* does not pass through all three points *P*,*Q*,*R*. But suppose that some other line *M* does pass through all three points. (assumption) (Make a new drawing.)

(5) Observe that points *P* and *Q* both lie on line *L* and also both lie on line *M*.

(6) Statement (5) contradicts something. (Explain the contradiction.)

Conclude that our assumption in step (4) was wrong. That is, there cannot be a line *M* that passes through all three points *P*,*Q*,*R*. Conclude that the points *P*,*Q*,*R* are non-collinear.

End of Proof

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[2] Justify the steps in the following proof of Incidence Geometry Theorem 3.

Incidence Geometry Theorem #3: In Incidence Geometry, there exist three lines that are not concurrent.

Proof:

Part I: Introduce three lines *L,M,N*.

(1) There exist three non-collinear points. We can call them *A,B,C*. (Justify.) (Make a drawing)

(2) There exists a unique line that passes through points *A* and *B*. We can call it *L*. (Justify.) (Make a new drawing)

(3) Line *L* does not pass through point *C*. (Justify.)

(4) Similarly, there exists a line *M* that passes through *B* and *C* and does not pass through *A*, and a line *N* that passes through *C* and *A* and does not pass through *B*. (Make a new drawing.)

Part II: Show that lines *L,M,N* are not concurrent.

(5) Suppose that lines *L,M,N* are concurrent. That is, suppose that there exists a point that all three lines *L,M,N* pass through. (Justify.)

(6) Any point that all three lines *L,M,N* pass through cannot be point *A, B,* or *C*. So the point that all three lines pass through must be a new point that we can call point *D*. (Justify.) (Make a new drawing)

(7) There are two lines that pass through points *A* and *D*. (Justify.)

(8) We have reached a contradiction. (Explain the contradiction.)

So our assumption in (5) was wrong. Lines *L,M,N* must be non-concurrent.
End of Proof
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