

Class Drill for Section 5.1 Justifying and illustrating Steps in proof of Theorem 27

Theorem 27: Given any line, each of its half-planes contains at least three non-collinear points.

Proof

(1) Given any line, call it L_1 . **(Make a drawing.)**

Introduce points A and B .

(2) There exist two distinct points on L_1 . Call them A and B . **(Justify.) (Make a new drawing.)**

Part I: Introduce Half-Plane H_C and show that it contains three non-collinear points.

(3) There exists a point not on L_1 . Call it C . **(Justify.) (Make a new drawing.)**

(4) Point C lies in one of the two half-planes determined by line L_1 . Call that half-plane H_C . **(Justify.) (Make a new drawing.)**

Introduce line L_2 .

(5) There exists a unique line passing through A and C . **(Justify.)**

(6) The line passing through A and C is not L_1 . So it must be new. Call it L_2 . **(Justify.) (Make a new drawing.)**

Introduce line L_3 .

(7) There exists a unique line passing through B and C . **(Justify.)**

(8) The line passing through B and C is not L_1 or L_2 . So it must be new. Call it L_3 . **(Justify.) (Make a new drawing.)**

Introduce point D .

(9) There exists a point such that $A * C * \text{Point}$. **(Justify.)**

(10) This point cannot be the same as any of our previous three points. So it must be a new point. Call it D .
So $A * C * D$. **(Justify.) (Make a new drawing.)**

(11) Point D is in half-plane H_C . **(Justify.) (Make a new drawing.)**

Introduce point E .

(12) There exists a point such that $B * C * \text{Point}$. **(Justify.)**

(13) This must be a new point. Call the new point E . So $B * C * E$. **(Justify.) (Make a new drawing.)**

(14) Point E is in half-plane H_C . **(Justify.) (Make a new drawing.)**

Conclusion of Part I:

(15) Points C and D and E are non-collinear. **(Justify.)**

(16) Conclusion: We have shown that there exist three non-collinear points C, D, E in half-plane H_C .