

Class Drill for Section 5.2 Justifying & Illustrating Steps in proof of Theorem 28 (Pasch's Theorem)

Theorem 28 (Pasch's Theorem) about a line intersecting a side of a triangle between vertices

If a line intersects the side of a triangle at a point between vertices, then the line also intersects the triangle at another point that lies on at least one of the other two sides.

Proof

- (1) Suppose that line L intersects side \overline{AB} of $\triangle ABC$ at a point D such that $A * D * B$.
- (2) Points A and B are on opposite sides of line L . (**Justify.**)

Let H_A and H_B be their respective half-planes.

- (3) Exactly one of the following statements is true. (**Justify.**)

(i) C lies on L . (**Make a drawing for case (i).**)

(ii) C is in H_A . (**Make a drawing for case (ii).**)

(iii) C is in H_B . (**Make a drawing for case (iii).**)

Case (i)

- (4) If C lies on L , then L intersects both \overline{AC} and \overline{BC} at point C . (**Justify.**)

Case (ii)

- (5) If C is in H_A , then points B and C lie on opposite sides of L . (**Justify.**)

- (6) In this case, L will intersect \overline{BC} at a point between B and C . (**Justify.**)

Case (iii)

- (7) If C is in H_B , then points A and C lie on opposite sides of L . (**Justify.**)

- (8) In this case, L will intersect \overline{AC} at a point between A and C . (**Justify.**)

Conclusion of cases

- (9) In every case, we see that L intersects \overline{AC} or \overline{BC} or both.

End of Proof

Class Drill for Section 5.2 Justifying & Illustrating Steps in proof of Theorem 29

Theorem 29 about a line intersecting two sides of a triangle between vertices

If a line intersects two sides of a triangle at points that are not vertices, then the line cannot intersect the third side.

Proof

(1) Suppose that line L intersects side of $\triangle ABC$ at a point D on side \overline{AB} and E on side \overline{AC} , where neither D nor E is a vertex. **(Make a drawing.)**

(2) Points A and B are on opposite sides of line L . **(Justify.)** Let H_A and H_B be their respective half-planes.

(3) Points A and C are on opposite sides of line L . **(Justify.)** Therefore, C is an element of half-plane H_B .

(4) Therefore, line L does not intersect \overline{BC} . **(Justify.)**

End of Proof