<u>Class Drill for Section 6.3: Every Angle has a Unique Bisector</u>

Theorem 40 says *Every Angle has a Unique Bisector*. Justify and illustrate the steps in the following proof. (Justifications may refer to any prior theorem and to Neutral Axioms <N1> through <N9> but not <N10>.)

Proof

(1) Suppose that angle $\angle ABC$ is given. **(Make a drawing.)**

Introduce special ray $\overrightarrow{BD}$ and show that it is a bisector of $\angle ABC$.

(2) The real number $m(\angle ABC)$ exists. **(Justify.)**

(3) Let H_C be the half-plane created by line $\overleftrightarrow{AB}$ that contains point C. Let $r = \frac{1}{2}m(\angle ABC)$. Observe ray $\overrightarrow{BA}$ lies on the edge of this half-plane, and that $0 < r < 90$ **(Justify the inequality) (Make a new drawing.)**

(4) There exists a ray $\overrightarrow{BD}$ such that $D \in H_C$ and $m(\angle ABD) = r$. **(Justify.) (Make a new drawing.)**

(5) Point D is in the interior of $\angle ABC$. (by statements (2), (4), and Theorem 39 II ➔ I) **(Make a new drawing.)**

(6) $m(\angle ABD) = m(\angle DBC)$. **(Justify. This will take 2 or 3 steps)**

(7) Ray $\overrightarrow{BD}$ is a bisector of $\angle ABC$. **(Justify.) (Make a new drawing.)**
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Show that ray $\overrightarrow{BD}$ is the only bisector of $\angle ABC$.

(8) Suppose that ray $\overrightarrow{BE}$ is a bisector of $\angle ABC$.

(9) Point E is in the interior of $\angle ABC$ and $m(\angle ABE) = m(\angle EBC)$. **(Justify.) (Make a new drawing.)**

(10) $m(\angle ABE) = \frac{1}{2} m(\angle ABC)$. **(Justify.)**

(11) Points E and C are on the same side of line $\overleftrightarrow{AB}$. That is point E is in half-plane H_C. **(Justify.) (Make a new drawing.)**

(12) Ray $\overrightarrow{BE}$ is the same ray as $\overrightarrow{BD}$. **(Justify.) (Make a new drawing.)**

End of Proof