

Class Drill for Section 6.4: The Linear Pair Theorem

Theorem 41 (The Linear Pair Theorem) says that if two angles form a linear pair, then the sum of their measures is 180. Justify and illustrate the steps in the following proof fragment. (Justifications may refer to any prior theorem and to Neutral Axioms <N1> through <N9> but not <N10>.)

Proof

- (1) Suppose that two angles form a linear pair.
- (2) The angles can be labeled $\angle ABD$ and $\angle DBC$ where $A * B * C$. **(Justify.) (Make a drawing.)**

- (3) Let $x = m(\angle ABD)$ and $y = m(\angle DBC)$. **(Update your drawing.)**

Proof Part 1: Show that $x + y$ cannot be less than 180.

- (4) Suppose that $x + y < 180$. (assumption)
- (5) Then $0 < x < x + y < 180$. **(Justify.)**

- (7) Let H_D be the half-plane created by line \overleftrightarrow{AC} that contains point D . **(Make a new drawing.)**

- (8) There exists a ray \overrightarrow{BE} such that $E \in H_D$ and $m(\angle ABE) = r$. **(Justify.) (Make a new drawing.)**

- (9) Observe that \overrightarrow{BC} and \overrightarrow{BE} are not the same ray (because E does not lie on line \overleftrightarrow{AB}).
- (10) Point D is in the interior of $\angle ABE$. **(Justify.)**

- (11) $m(\angle ABD) + m(\angle DBE) = r$. **(Justify.)**

(12) Therefore, $m(\angle DBE) = y$. **(Make a new drawing.)**

(13) Ray \overrightarrow{BD} intersects segment \overline{AE} at a point between A and E . **(Justify.)** We can label the point of intersection F . **(Make a new drawing.)**

(14) Points A and E are on opposite sides of line \overleftrightarrow{BD} . **(Justify.)**

(15) Points A and C are on opposite sides of line \overleftrightarrow{BD} . **(Justify)**

(16) Points E and C are on the same side of line \overleftrightarrow{BD} . **(Justify.)**

(17) Let H_C be the half-plane created by line \overleftrightarrow{BD} that contains point C . Observe that this half-plane also contains point E . **(Make a new drawing.)**

(18) Rays \overrightarrow{BC} and \overrightarrow{BE} have $C \in H_C$ and $E \in H_C$ and $m(\angle DBC) = y$ and $m(\angle DBE) = y$ but they are not the same ray. (by steps (17), (3), (12), (9)). **(Make a new drawing.)**

(19) Statement (18) is a contradiction. **(What does it contradict?)** Therefore, our assumption in step (4) was incorrect. That is, $x + y$ cannot be less than 180.

End of Proof Part 1 showing that $x + y$ cannot be less than 180.