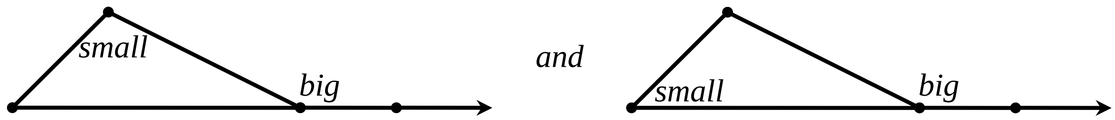
Class Drill for Section 7.3: The Neutral Exterior Angle Theorem

Justify the steps in the proof of the following theorem:

Theorem 59 Neutral Exterior Angle Theorem

In Neutral Geometry, the measure of any exterior angle is greater than the measure of either of its remote interior angles.

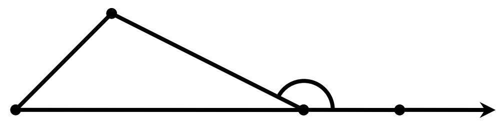
Remark: The statement of the theorem can be illustrated by the picture below.



Theorem 59

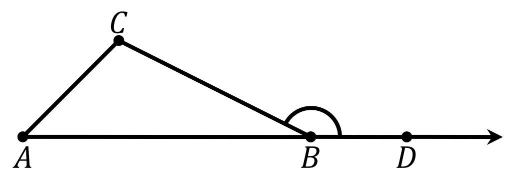
Proof

(1) Suppose that a triangle and an exterior angle are given.



Part I: Show that the measure of the given exterior angle is larger than the measure of the remote interior angle that the exterior point does not lie on.

(2) Label the points so that the triangle is ΔABC and the exterior angle is $\angle CBD$. The two remote interior angles are $\angle BAC$ and $\angle BCA$. Observe that point D lies on side $\overrightarrow{AB}$ of $\angle BAC$ but point D does not lie on either of the sides of angle $\angle BCA$. Our goal in Part I of the proof will be to show that $m(\angle CBD) > m(\angle BCA)$.



(3) There exists a point E that is the midpoint of side $\overline{BC}$. **(Justify.) (Make a drawing.)**

(4) $\overline{EB} \cong \overline{EC}$. **(Justify.) (Update your drawing.)**

(5) There exists a point F such that $A * E * F$. **(Justify.) (Make a new drawing.)**

(6) There exists a point G on ray $\overrightarrow{EF}$ such that $\overline{EG} \cong \overline{EA}$. **(Justify.) (Update your drawing.)**

(7) $\angle AEC \cong \angle GEB$. **(Justify.) (Make a new drawing.)**

(8) $\Delta AEC \cong \Delta GEB$. **(Justify.) (Make a new drawing.)**

Make observations about angles

(9) $\angle ACE \cong \angle GBE$. **(Justify.) (Make a new drawing.)**

Prove that Point G is in the interior of $\angle CBD$

(10) Points E and G are on the same side of line $\overleftrightarrow{BD}$. **(Justify.) (Make a new drawing.)**

(11) Points E and C are on the same side of line $\overleftrightarrow{BD}$. **(Justify.) (Make a new drawing.)**

(12) Therefore, points C and G are on the same side of line $\overleftrightarrow{BD}$. **(Justify.) (Make a new drawing.)**

(13) Points A and G are on opposite sides of line $\overleftrightarrow{BC}$. **(Justify.) (Make a new drawing.)**

(14) Points A and D are on opposite sides of line $\overleftrightarrow{BC}$. **(Justify.) (Make a new drawing.)**

(15) Therefore, points G and D are on the same side of line $\overleftrightarrow{BC}$. **(Justify.) (Make a new drawing.)**

(16) Conclude that point G is in the interior of $\angle CBD$. **(Justify.) (Make a new drawing.)**

Make some more observations about angles

(17) $m(\angle CBD) > m(\angle CBG)$. **(Justify.)**

(18) $m(\angle CBD) > m(\angle BCA)$. **(Justify.)**

Part II: Show that the measure of the given exterior angle is also larger than the measure of the remote interior angle that the exterior point does lie on.

(19) There exists a point H such that $C * B * H$. **(Justify.) (Make a new drawing.)** Observe that $\angle ABH$ is an exterior angle for ΔABC and also observe that the point H does not lie on the remote interior angle $\angle BAC$.

(20) $m(\angle ABH) > m(\angle BAC)$ (by statements identical to statements (2) through (12), but with points A, C, D replaced in all statements with points C, A, H.)

(21) $m(\angle ABH) = m(CBD)$. **(Justify.) (Make a new drawing.)**

(22) $m(\angle CBD) > m(\angle BAC)$. **(Justify.)**

End of proof