

## Class Drill for Section 7.7: The Alternate Interior Angle Theorem

Theorem 74 The Alternate Interior Angle Theorem for Neutral Geometry

**Given:** Neutral Geometry, lines  $L$  and  $M$  and a transversal  $T$

**Claim:** If a pair of alternate interior angles is congruent, then lines  $L$  and  $M$  are parallel.

**Contrapositive:** If  $L$  and  $M$  are not parallel, then a pair of alternate interior angles are not congruent.

**Proof (Indirect proof by method of contraposition)**

(1) Suppose that in Neutral Geometry, lines  $L$  and  $M$  and a transversal  $T$  are given, and that  $L$  and  $M$  are not parallel. **(make a drawing)**

(2) Let  $A$  be the point of intersection of lines  $L$  and  $M$ , let  $B$  be the point of intersection of lines  $L$  and  $T$ , and let  $C$  be the point of intersection of lines  $M$  and  $T$ . **(update drawing)**

(3) There exists a point  $D$  such that  $A * B * D$ . **(Make a new drawing)**

(4) Observe that  $\angle CBD$  is an exterior angle for  $\triangle ABC$ , and  $\angle BCA$  is one of its remote interior angles. **(Make a new drawing)**

(5)  $m(\angle CBD) > m(\angle BCA)$ . **(Justify.) (Make a new drawing)**

(6) Observe  $\angle CBD$  and  $\angle BCA$  are alternate interior angles and they are not congruent. That is, lines  $L, M, T$  do not have the special angle property.

**End of Proof**