

Class Drill for Section 8.1: Theorem 78 about Tangent Lines and Radial Segments

Theorem 78 In Neutral Geometry, tangent lines are perpendicular to the radial segment.

Given: A segment \overline{AB} and a line L passing through point B .

Claim: The following statements are equivalent.

(i) Line L is perpendicular to segment \overline{AB} .

(ii) Line L is tangent to $\text{Circle}(A, AB)$ at point B . That is, L only intersects $\text{Circle}(A, AB)$ at point B .

Proof that (i) \rightarrow (ii)

(1) Suppose that (i) is true. That is, suppose that line L is perpendicular to segment \overline{AB} . **(Make a drawing.)**

(2) Let C be any point on line L except B . **(Make a new drawing.)**

(3) Observe that points A, B, C form a triangle with right angle at B . Segment \overline{AC} is the hypotenuse of this triangle. So $AC > AB$. Therefore C is in the exterior of $\text{Circle}(A, AB)$. **(Make a new drawing.)**

(4) Conclude that line L intersects $\text{Circle}(A, AB)$ at point B , but does not intersect the circle at any other point. That is, line L is tangent to $\text{Circle}(A, AB)$ at point B . So (ii) is true.

Proof that \sim (i) \rightarrow \sim (ii) (Notice that this is the contrapositive of (ii) \rightarrow (i).)

(5) Suppose that (i) is false. That is, suppose that line L is not perpendicular to segment \overline{AB} .

(6) There are two possibilities:

(a) Line L is the same line as line \overleftrightarrow{AB} .

(b) Line L is not the same line as line \overleftrightarrow{AB} .

Case (a)

(7) Suppose that line L is the same line as line \overleftrightarrow{AB} . **(Make a new drawing.)**

(8) There exists a point C such that $C * A * B$ and such that $\overline{AC} \cong \overline{AB}$. **(Justify) (Make a new drawing.)**

(9) Points B and C are both on $\text{Circle}(A, AB)$. **(Justify.) (Make a new Drawing.)**

(10) Conclude that line L intersects $\text{Circle}(A, AB)$ at more than one point. That is, line L is not tangent to $\text{Circle}(A, AB)$. So, Statement (ii) is false in this case.

Case (b)

(11) Suppose that line L passes through point B and is not perpendicular to line \overleftrightarrow{AB} and is not the same line as line \overleftrightarrow{AB} . (**Make a new drawing.**)

(12) There exists a line M that passes through point A and is perpendicular to L . Let C be the point of intersection of lines L and M . (**Make a new drawing.**)

(13) There exists a point D such that $B * C * D$ and $\overline{CD} \cong \overline{CB}$. (**Make a new drawing.**) (**Justify.**)

(14) Observe that $\triangle ACD \cong \triangle ACB$. (**Justify.**)

(15) Therefore, $\overline{AD} \cong \overline{AB}$. (**Justify.**) So point D also lies on the circle. (**Make a new drawing.**)

(16) Conclude that L intersects the circle at more than one point. That is, line L is not tangent to the circle. So Statement (ii) is false in this case as well.

Conclusion of Cases

(17) Conclude that statement (ii) is false in either case.

End of Proof