

Class Drill for Section 8.4 Points on Angle Bisector are Equidistant from Sides of Angle

Justify the steps in the proof of the following theorem:

Theorem 91 about points on the bisector of an angle in Neutral Geometry

Given: Neutral Geometry, angle $\angle BAC$, and point D in the interior of the angle

Claim: The following statements are equivalent

- (i) D lies on the bisector of angle $\angle BAC$.
- (ii) D is equidistant from the sides of angle $\angle BAC$.

Proof

- (1) In Neutral Geometry, suppose that point D lies in the interior of angle $\angle BAC$. (**Make a drawing.**)

Proof that (i) \rightarrow (ii)

- (2) Suppose that (i) is true. That is, suppose that D lies on the bisector of angle $\angle BAC$. (**Make a new drawing.**)

- (3) Let point E be the foot of the perpendicular from D to line \overleftrightarrow{AB} , and let point F be the foot of the perpendicular from D to line \overleftrightarrow{AC} . (**Make a new drawing.**)

- (4) Then $\triangle DAE \cong \triangle DAF$. (**Justify.**) (**Make a new drawing.**)

- (5) So $\overline{DE} \cong \overline{DF}$. (**Justify.**) (**Make a new drawing.**)

- (6) Conclude that D is equidistant from the sides of angle $\angle BAC$. That is, (ii) is true.

Proof that (ii) \rightarrow (i)

(7) Suppose that (ii) is true. That is, suppose that D is equidistant from the sides of angle $\angle BAC$. **(Make a new drawing.)**

(8) Let point E be the foot of the perpendicular from D to line \overleftrightarrow{AB} , and let point F be the foot of the perpendicular from D to line \overleftrightarrow{AC} . **(Make a new drawing.)**

(9) Then $\overline{DE} \cong \overline{DF}$. **(Justify.) (Make a new drawing.)**

(10) Then $\triangle DAE \cong \triangle DAF$. **(Justify.) (Make a new drawing.)**

(11) So $\angle DAE \cong \angle DAF$. **(Justify.) (Make a new drawing.)**

(12) Conclude that D lies on the bisector of angle $\angle BAC$. That is, (i) is true.

End of Proof