Class Drill for Section 8.4 Concurrence of Angle Bisectors of a Triangle

Theorem 92 In Neutral Geometry, the three angle bisectors of any triangle are concurrent at a point that is equidistant from the three sides of the triangle.

Proof

(1) In Neutral Geometry, suppose that triangle ΔABC is given. **(Make a drawing.)**

Show that the bisectors of $\angle A$ and $\angle B$ intersect.

(2) There exists a ray $\overrightarrow{AD}$ that bisects $\angle CAB$. **(Justify.) (Make a new drawing.)**

(3) Point D lies in the interior of angle $\angle CAB$. **(Justify.) (Make a new drawing.)**

(4) Ray $\overrightarrow{AD}$ intersects side $\overline{BC}$ at a point that we can call E. **(Justify.) (Make a new drawing.)**

(5) There exists a ray $\overrightarrow{BF}$ that bisects $\angle ABE$. **(Justify.) (Make a new drawing.)**

(6) Point F lies in the interior of angle $\angle ABE$. **(Justify.) (Make a new drawing.)**

(7) Ray $\overrightarrow{BF}$ intersects segment $\overline{AE}$ at a point that we can call G. **(Justify.) (Make a new drawing.)** We have shown that the bisectors of $\angle A$ and $\angle B$ intersect at G.

Consider distances from the point of intersection to the sides of the triangle

(8) The distance from G to line $\overleftrightarrow{AC}$ equals the distance from G to line $\overleftrightarrow{AB}$. **(Justify.) (Make a new drawing.)**

(9) The distance from G to line $\overleftrightarrow{BA}$ equals the distance from G to line $\overleftrightarrow{BC}$. **(Justify.) (Make a new drawing.)**

(10) So the distance from G to line $\overleftrightarrow{CA}$ equals the distance from G to line $\overleftrightarrow{CB}$. **(Justify.) (Make a new drawing.)**

(11) Therefore, point G lies on the bisector of $\angle BCA$. **(Justify.) (Make a new drawing.)**

(12) We have shown that the bisectors of all three angles $\angle A$ and $\angle B$ and $\angle C$ intersect at G and that G is equidistant from the three sides of the triangle.

End of Proof