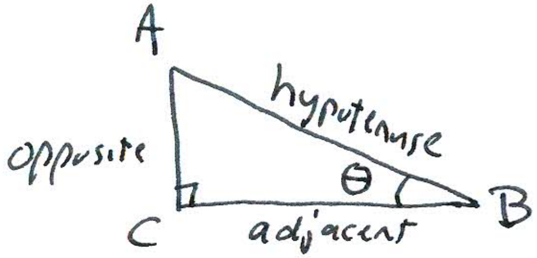
MATH 2110 (Barsamian) Day #28 (Fri Nov 1, 2019)

Recall Trig Ratios from Section 6.4



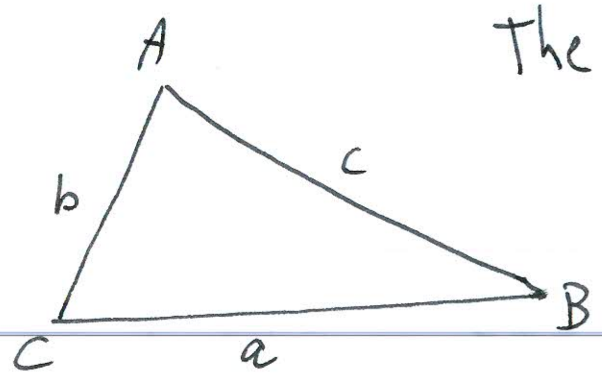
for a right triangle

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\text{opposite}}{\text{adjacent}}$$

In Section 6.5 we learn two new facts valid for any triangle ΔABC.



The Law of Sines: $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

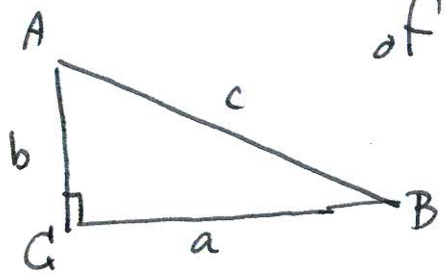
The Law of Cosines: $a^2+b^2-2ab\cos(C) = c^2$

Similarly for b, c, A, a

and for c, a, B, b

Observe special cases of the law of cosines.

Suppose $C = 90°$. Then $\cos(C) = \cos(90) = 0$. In this case, the law

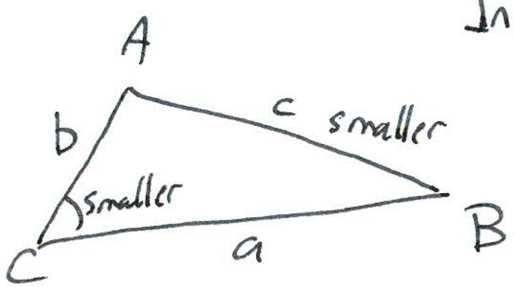


of cosines becomes $a^2+b^2-\underbrace{2ab\cancelto{0}{\cos(90)}}_{\text{zero}} = c^2$

$a^2+b^2 = c^2$ Pythagorean Theorem

Suppose $C < 90$ Then cos(C) will satisfy $0 < \cos(C) < 1$

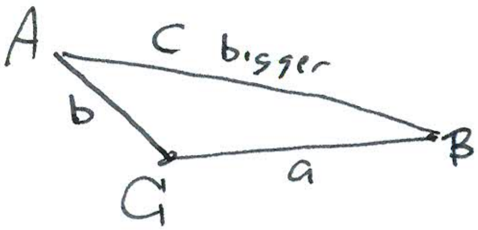
In this case, $c^2 = a^2+b^2-\underbrace{2ab\underbrace{\cos(C)}_{\text{positive}}}_{\text{positive}} < a^2+b^2$



So when the angle C gets smaller, the law of cosines tells us that side c will also get smaller

Suppose $C > 90$, Then we will have $-1 < \cos(C) < 0$.

In this case, $c^2 = a^2+b^2-\underbrace{2ab\underbrace{\cos(C)}_{\text{negative}}}_{\text{negative}} > a^2+b^2$



So when the angle C gets bigger, the law of Cosines tells us that side C will also get bigger.

Example #1 (Problem 6.5 #2) $\triangle ABC$ has sides a, b, c.
Given $\angle A = 30°$, $\angle C = 40°$, and $a = 20$, Use the law of Sines to find C.
(exact answer then decimal approximation.)

Solution

$$\frac{\sin(A)}{a} = \cancel{\sin B}\ \frac{\sin(C)}{c}$$

$$c = \frac{a\sin(C)}{\sin(A)} = \frac{20\sin(40)}{\sin(30)} \approx 25.712$$

exact approx

A	30
B	
C	40
a	20
b	
c	≈ 25.712

Example #3 (Problem 6.5 #8) ΔABC has sides a, b, c

Given $\angle B = 75$, $b = 20$, $c = 5$, use the law of sines to find C.

(exact and approximate answers)

Solution

$$\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$\sin(C) = \frac{c\sin(B)}{b}$$

$$C = \sin^{-1}\left(\frac{c\sin(B)}{b}\right) = \sin^{-1}\left(\frac{5\sin(75)}{20}\right) \approx 13.97^\circ$$

exact approx

A

$B = 75$

$C \approx 13.97$

a

$b = 20$

$c = 5$

Example #3 (6.5 #12) $\triangle ABC$ has sides a, b, c

Given $\angle B = 50$ and $\angle C = 60$ and $c = 10$, use the Law of Sines to find the measures of all remaing parts of $\triangle ABC$.
(This is called "solving the triangle")

Solution Observe that by triangle angle sum, we know $A = 70$

A	70
B	50
C	60
a	≈ 10.851
b	≈ 8.846
c	10

$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}$$

$$a = c\,\frac{\sin(A)}{\sin(C)} = \underbrace{10\,\frac{\sin(70)}{\sin(60)}}_{\text{exact}} \approx \underbrace{10.851}_{\text{approximate}}$$

$$\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$b = c\,\frac{\sin(B)}{\sin(C)} = \underbrace{10\,\frac{\sin(50)}{\sin(60)}}_{\text{exact}} \approx \underbrace{8.846}_{\text{approx}}$$