

Group Work: Using Similarity to prove the Pythagorean Theorem

Theorem: The Pythagorean Theorem of Euclidean Geometry

In Euclidean Geometry, the sum of the squares of the length of the two sides of any right triangle equals the square of the length of the hypotenuse. That is, in Euclidean Geometry, given triangle $\triangle ABC$ with $a = BC$ and $b = CA$ and $c = AB$, if angle $\angle C$ is a right angle, then $a^2 + b^2 = c^2$.

Proof

(1) In Euclidean Geometry, suppose that triangle $\triangle ABC$ has a right angle at C and that $a = BC$ and $b = CA$ and $c = AB$. **(Make a drawing.)**

(2) Let D be the foot of the altitude drawn from vertex C . That is, D is the point on side \overline{AB} such that segment \overline{CD} is perpendicular to side \overline{AB} . Let $x = AD$ and $y = BD$. **(Make a new drawing.)**

(3) $\triangle ADC \sim \triangle ACB$. **(Justify) (Make a new drawing.)**

(4) $\frac{x}{b} = \frac{b}{c}$. **(Justify)**

(5) $cx = b^2$. **(Justify)**

(6) $\triangle BDC \sim \triangle BCA$. **(Justify) (Make a new drawing.)**

(7) $\frac{y}{a} = \frac{a}{c}$. **(Justify)**

(8) $cy = a^2$. **(Justify)**

(9) $a^2 + b^2 = cx + cy$. **(Justify)**

(10) $a^2 + b^2 = c(x + y)$. **(arithmetic)**

(11) $a^2 + b^2 = c^2$. **(Justify)**

End