Postulates and Theorems

POSTULATE 2.1 Every Line contains at least two distinct points

POSTULATE 2.2 Two points are contained in one and only one line

POSTULATE 2.3 If two points are in a plane, then the line containing these points is also in the plane.

POSTULATE 2.4 Three noncollinear points are contained in one and only one plane, and every plane contains at least three non collinear points.

POSTULATE 2.5 In space, there exist at least four points that are not all coplanar.

POSTULATE 2.6 The Ruler Postulate: Every line can be made into an exact copy of the real number line using a 1-1 correspondence.

POSTULATE 2.7 The Protractor Postulate, summarized: Angle measure in our abstract geometry behaves like angle measure obtained by using a protractor in a drawing. In a drawing, we place the angle vertex at the midpoint of the bottom edge, and place one ray of the angle on the 0° mark. The number by the second ray of the angle is the measure of the angle. It can be a number between 0° and 180° inclusive. As part of this, we know that if two angles form a *linear pair*, then their angle measures will add up to 180°.

Theorem 3.8 Pythagorean Theorem: Given a triangle ΔABC with opposite sides a, b, c.
If angle C is a right angle, then the equation $a^2 + b^2 = c^2$ is true.

Theorem 4.1 The Vertical Angle Theorem (The Bow Tie Angle Theorem):
Vertical angles (angles that form a bow tie) are congruent.

POSTULATE 4.1 *SAS* Congruence: If two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of another triangle, then the two triangles are congruent.

Theorem 4.3 *HL* Congruence: If the hypotenuse and leg of one right triangle are congruent respectively to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.

POSTULATE 4.2 *ASA* Congruence: If two angles and the included side of one triangle are congruent respectively to two angles and the included side of another triangle, then the two triangles are congruent.

POSTULATE 4.3 *SSS* Congruence: If three sides of one triangle are congruent respectively to three sides of another triangle, then the two triangles are congruent.

Theorem 4.4 Converse of the Pythagorean Theorem. Given a triangle ΔABC with opposite sides a, b, c.
If the equation $a^2 + b^2 = c^2$ is true, then angle C is a right angle.

Theorem 4.5 *CS* ⇒ *CA*: In any triangle, if two sides are congruent, then the opposite angles are also congruent.

Theorem 4.7 *CA* ⇒ *CS*: In any triangle, if two angles are congruent, then the opposite sides are also congruent.

Theorem 4.9 (about special rays in isosceles triangles) Given triangle ΔABC with $\overline{BA} \cong \overline{BC}$ and some point D on side $\overline{BC}$. The following are equivalent. (That is, they are either all true or all false.)

(1) Ray $\overrightarrow{BD}$ bisects angle $\angle ABC$. That is $\angle ABD \cong \angle CBD$.

(2) Ray $\overrightarrow{BD}$ bisects side $\overline{AC}$. That is $\overline{AD} \cong \overline{CD}$.

(3) Ray $\overrightarrow{BD}$ is perpendicular to side $\overline{AC}$. That is $\overline{BD} \perp \overline{AC}$.

Theorem 4.10 Perpendicular Bisector Theorem: A point is on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the line segment.

Theorem 5.1 The Alternate Interior Angle Theorem:
Given lines L, M cut by transversal T, if alternate interior angles are congruent then $L \parallel M$.

Corollary 5.3 The Corresponding Angle Theorem
Given lines L, M cut by transversal T, if corresponding angles are congruent then $L \parallel M$.

Corollary 5.4 The Supplementary Interior Angle Theorem
Given lines L, M cut by transversal T,
If interior angles on the same side of the transversal are *supplementary* (if their sum is 180), then $L \parallel M$.

Postulate 5.1 The Parallel Postulate:
Given a line L and a point P not on L, there is exactly one line M such that M passes through P and $M \parallel L$.

Theorem 5.5 The Converse of the Alternate Interior Angle Theorem:
Given lines L, M cut by transversal T. If $L \parallel M$, then alternate interior angles are congruent.

Corollary 5.7 The Converse of the Corresponding Angle Theorem
Given lines L, M cut by transversal T. If $L \parallel M$, then corresponding angles are congruent.

Corollary 5.8 The Converse of the Supplementary Interior Angle Theorem
Given lines L, M cut by transversal T.
If $L \parallel M$, then interior angles on the same side of the transversal are *supplementary* (their sum is 180).

Theorem 5.9 Angle Measure in a Triangle: Triangle angle sum is 180

Corollary 5.10 The Euclidean Exterior Angle Theorem: In any triangle, the measure of an exterior angle is equal to the sum of the measures of the non-adjacent interior angles.

Corollary 5.11 *AAS* Congruence: If two angles and a side of one triangle are congruent respectively to two angles and a side of another triangle, then the two triangles are congruent.

Theorem 5.13 The Angle Bisector Theorem: A point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.

Theorems 5.15 through 5.21 are part of (but not all of) the following **MEGA THEOREM**.

The Six Equivalent Statements Theorem: (Six Equivalent Statements about Convex Quadrilaterals)
Given any convex quadrilateral,
the following six statements are equivalent (TFAE). That is, they are either all true or all false.
(i) Both pairs of opposite sides are parallel. That is, the quadrilateral is a parallelogram.
(ii) Both pairs of opposite sides are congruent.
(iii) One pair of opposite sides is both congruent and parallel.
(iv) Each pair of opposite angles is congruent.
(v) Either diagonal creates two congruent triangles.
(vi) The diagonals bisect each other.

Theorem 5.23 The diagonals of a rhombus are perpendicular to each other.

Theorem 5.24 If the diagonals of a parallelogram are perpendicular to each other, then the parallelogram is a rhombus.

Theorem 5.25 A parallelogram is a rhombus if and only if its diagonals bisect its vertex angles.

Theorem 5.27 If a parallelogram has one right angle, then it is a rectangle.

Theorem 5.28 The diagonals of a rectangle are congruent.

Theorem 5.29 If a parallelogram has congruent diagonals, then it is a rectangle.

Theorem 5.30 The base angles of an isosceles trapezoid are congruent.

Theorem 5.31 If the base angles of a trapezoid are congruent, then the trapezoid is isosceles.

Theorem 5.32 Given three parallel lines cut by two transversals.
If the three parallel lines cut two congruent segments in one of the transversals,
then the three parallel lines also cut two congruent segments in the other transversal.

Theorem 6.4 *AA* Similarity: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem 6.6 *SAS* Similarity: If two sides of one triangle are proportional, respectively to two sides of another triangle and if the included angles are congruent, then the triangles are similar.

Theorem 6.8 *SSS* Similarity: If three sides of one triangle are proportional, respectively, to three sides of another triangle, then the triangles are similar.

Theorem 6.10 Side Splitting: If a line cuts through two sides of a triangle and is parallel to the third side, then the line creates a smaller triangle that is similar to the larger triangle. (As a result, there are some ratios that are equal, but I will leave it to you to figure out which ratios.)

Theorem 6.11 Triangle Midsegment Theorem:
A midsegment of a triangle is parallel to the third side and is half as long as the third side.

Corollary 6.12 Midquad Theorem:
If ABCD is a convex quadrilateral and EFGH are the midpoints of the sides, then EFGH is a parallelogram.

Theorem 6.15 The Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Theorem 6.16 The Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$.

Theorem 7.1 The Inscribed Angle Theorem:
The measure of an inscribed angle in a circle is equal to half the measure of its intercepted arc.

Corollary 7.2 Inscribed angles that intercept the same arc (or congruent arcs) are congruent.

Corollary 7.3 An inscribed angle is inscribed in a semicircle if and only if it is a right angle.

Theorem 7.4 The perpendicular bisector of a chord contains the center of the circle.

Corollary 7.5 The intersection of the perpendicular bisectors of any two non-parallel chords of a circle is the center of the circle.

Corollary 7.6 If two circles intersect in two points A, B then the line containing the centers of the two circles is the perpendicular bisector of segment $\overline{AB}$.

Theorem 7.7 If two chords intersect at a point in the interior of a circle, then the measure of any one of the vertical angles formed is equal to half the sum of the measures of the two arcs intercepted by the two vertical angles.

Theorem 7.8 If one chord has endpoints A, B and another chord has endpoints C, D and if the two chords intersect at a point E inside the circle, then $AE \cdot BE = CE \cdot DE$.

Theorem 7.9 If two secants intersect at a point outside a circle, then the measure of the acute angle formed by the two secants is equal to half the difference of the measures of the intercepted arcs.

Theorem 7.10 If one secant intersects a circle at points A, B and another secant intersects a circle at points C, D and if the two secants intersect at a point E outside the circle, then $AE \cdot BE = CE \cdot DE$.

Theorem 7.11 Given a circle centered at point O and a line $\overleftrightarrow{AB}$ that intersects the circle at point A, the following are equivalent. (That is, they are either both true or both false.)

(1) Line $\overleftrightarrow{AB}$ is tangent to the circle at point A.

(2) Line $\overleftrightarrow{AB}$ is perpendicular to radial segment $\overline{OA}$.

Theorem 7.12 The measure of any angle formed when a chord intersects a tangent line at the point of tangency is equal to half the measure of the arc intercepted by the chord and the tangent line.

Theorem 7.13 If a secant and a tangent line intersect outside the circle, then the measure of the angle formed by the two lines is equal to half of the difference in measure of the larger and smaller intercepted arcs.

Theorem 7.14 If two tangent lines intersect outside the circle, then the measure of the angle formed by the two lines is equal to half of the difference in measure of the larger and smaller intercepted arcs.

Corollary 7.15 If two tangent lines are drawn to a circle from the same point in the exterior of the circle, then the distances from the common point to the points of tangency are equal.

Theorem 7.16 If a line is tangent to a circle at point A and a secant intersects the circle at points C, D and if the two lines intersect at a point E outside the circle, then $AE \cdot AE = CE \cdot DE$.

Theorem 7.19 The angle bisectors of a triangle intersect in a single point, called the *incenter*.

Theorem 7.10 The medians of a triangle intersect in a single point, called the *centroid*. The centroid is two-thirds of the way from anyh vertex to the other endpoint of the median from that vertex.