ClassActivity #1 for Section 4.1: Justifying and illustrating steps in a proof about half-planes

(Exercise 4.1#4) Given: a line in a metric geometry that satisfies the Plane Separation Axiom
Claim: Each of the half-planes determined by L contains a point.
Proof

(1) Suppose L is a line in a metric geometry that satisfies the Plane Separation Axiom. (Make a drawing.)

(2) There exist two distinct points on L. Call them P and Q. (Justify.) (Make a new drawing.)

(3) There exists a point not on L. Call it R. (Justify.) (Make a new drawing.)

(4) Point R lies in one of the two half-planes determined by line L. (Justify.) Call that half-plane H_1. (Make a new drawing.)

(5) There exists a unique line passing through P and R. (Justify.)

(6) The line passing through P and R is not L. So it must be new. Call it M. (Justify.) (Make a new drawing.)

(7) There exists a point such that $R - P - point$. (Justify.)

(8) This point cannot be the same as any of our previous three points. (Justify.) So it must be a new point. Call it S. So $R - P - S$. (Make a new drawing.)

(9) Point S lies in the other half-plane determined by line L. (Justify.) Call that half-plane H_2.

(10) We have shown that half-planes H_1, H_2 each contain a point. (Make a new drawing.)

End of Proof

ClassActivity #2 for Section 4.1: Justifying and illustrating steps in a proof about half-planes

(Exercise 4.1#5) Given: a line in a metric geometry that satisfies the Plane Separation Axiom
Claim: Each of the half-planes determined by L contains three non-collinear points.
Proof

(1) Suppose L is a line in a metric geometry that satisfies the Plane Separation Axiom, and suppose that H is one of the half-planes determined by L. (Make a drawing.)

(2) There exist two distinct points on L. Call them A and B. (Justify.) (Make a new drawing.)

(3) There exists a point C in H. (Justify.) (Make a new drawing.)

(4) There exists a unique line passing through A and C. (Justify.)

(5) The line passing through A and C is not L. (Justify.) So it must be new. Call it M. (Make a new drawing.)

(6) There exists a unique line passing through B and C. (Justify.)

(7) The line passing through B and C is not L or M. (Justify.) So it must be new. Call it N. (Make a new drawing.)

(8) There exists a point such that $A - C - point$. (Justify.)

(9) This point cannot be the same as any of our previous three points. (Justify.) So it must be a new point. Call it D. So $A - C - D$. (Make a new drawing.)

(10) Point D is in half-plane H. (Justify.) (Make a new drawing.)

(11) There exists a point such that $B - C - point$. (Justify.)

(12) This point cannot be the same as any of our previous three points. (Justify.) So it must be a new point. Call it E. So $B - C - E$. (Justify.) (Make a new drawing.)

(13) Point E is in half-plane H. (Justify.) (Make a new drawing.)

(14) Points C, D, E are non-collinear. (Justify.) And notice that all three lie in half-plane H.

(15) We have not said which of the two half-planes determined by L we are talking about. So all that we have said holds for both half-planes. That is, each of the half-planes contains three non-collinear points.

End of Proof.