

Proving Statements Using Given Conditional Statements and their Contrapositives

Suppose that two axioms (or theorems) are stated in the form of conditional statements, as follows.

- Axiom <100>: If the dog is blue, then the car is red.
- Axiom <101>: If the car is red, then the bear is hungry.

The contrapositives of these two axioms would be the following statements:

- Axiom <100> (contrapositive): If the car is not red, then the dog is not blue.
- Axiom <101> (contrapositive): If the bear is not hungry, then the car is not red.

Remember that the contrapositive statements are logically equivalent to the original statements.

Suppose that we wanted to prove that the car is red. Then clearly, we would use Axiom <100>. Our strategy would be to first prove somehow that the dog is blue, and then use Axiom <100> to say that the car is red. Note that we would not use Axiom <101> to prove that the car is red. Axiom <101> tells us something about the situation where we already know that the car is red. (It tells us that in this situation, the bear is hungry.)

Now, suppose that we wanted to prove that the car was not red. It is important to realize that Axiom <100> does not help us in this case! If we want to prove that the car is not red, then we need to use Axiom <101> (contrapositive). Our strategy would be to first prove somehow that the bear is not hungry, and then use Axiom <101> (contrapositive) to say that the car is not red.

This discussion is relevant to your use of PSA (ii) and PSA (iii) in proofs.

- PSA (ii) If P, Q are in the same half-plane, then line L does not intersect segment \overline{PQ}
- PSA (ii) (contrapositive) If line L does intersect segment \overline{PQ} , then P, Q are not in the same half-plane.
- PSA (iii) If P, Q are not in the same half-plane, then line L intersects segment \overline{PQ}
- PSA (iii) (contrapositive) If line L does not intersect segment \overline{PQ} , then P, Q are in the same half-plane.

For instance, suppose that you know that two points P and Q are not on some line L , and you want to prove that they are in the same half plane of L .

You should not use PSA (ii). That statement says something about the situation where you already know that points P and Q are in the same half plane. (It says that in that situation, segment \overline{PQ} does not intersect line L .)

Rather, you should use PSA (iii) (contrapositive). Your strategy should be to prove somehow that segment \overline{PQ} does not intersect line L , and then use PSA (iii) (contrapositive) to say that points P and Q are in the same half plane.

Or instead, suppose that you know that two points P and Q are not on some line L , and you want to prove that they are *not* in the same half plane of L .

You should not use PSA (iii). That statement says something about the situation where you already know that points P and Q are not in the same half plane. (It says that in that situation, segment \overline{PQ} does intersect line L .)

Rather, you should use PSA (ii) (contrapositive). Your strategy should be to prove somehow that segment \overline{PQ} does intersect line L , and then use PSA (ii) (contrapositive) to say that points P and Q are not in the same half plane.