Discussion Session Sept 16, 2020

Conceptual Questions

[1] Is every integer either even or odd?

Townsend: yes

Answer: yes

Definition of Even and Odd Numbers

Words: n *is even*

Meaning: $\exists k \in \boldsymbol{Z}(n = 2k)$

Words: n *is odd*

Meaning: $\exists k \in Z(n = 2k + 1)$

We'll learn a theorem (The Parity Theorem) that says that every integer is either even or odd (exclusive or)

[2] Is every integer either prime or composite?

Blake: 1 is neither, so no

Definition of Composite Numbers

Words: *n is composite*

Meaning: $\exists r, s \in \boldsymbol{Z}\big((r > 1) \wedge (s > 1) \wedge (n = rs)\big)$

Definition of Prime Numbers

Words: *n is prime*

Meaning: $(n \in \boldsymbol{Z}) \wedge (n > 1) \wedge (n$ is not *composite*$)$

The union

Composite Numbers $\cup$ Prime numbers

is just the set of integers > 1.

Question about "Proof Structure"

[3] Here is a proof frame

Proof

(1) Suppose that m is an odd integer and n is an even integer.

$\vdots$ some steps here

(6) Therefore $m^2 + n^2$ is odd.

End of Proof

Questions

(a) What is the statement that is being proved?

(b) What will have to be stated in steps (2) and (3)?

(c) What will have to be stated in step (5)?

(a) Townsend

$\forall$ odd integer m, even integer n (m^2+n^2 is odd)

Universal statement

Universal Conditional

$\forall$ integers m, n (If m is odd and n is even then m^2+n^2 is odd)

(b) Blake

(2) There exists an integer k such that $m=2k+1$ (by 1 and definition of odd)

Kelsie

(3) There exists an integer j such that $n=2j$ (by 1 and definition of even)

(c) Tinxiao

(5) There exists an integer p such that $m^2+n^2=2p+1$ (justified somehow)

(6) m^2+n^2 is odd (by 5 and the definition of odd)

[4] Here is a statement and a proposed proof. Comment on the validity or invalidity.

universal

Statement S: For every integer n, the number $n^2 - n + 11$ is prime.

(False statement)

domain is infinite

Predicate $P(n)$

Proof

invalid proof ✗

1) With $n = 1$: the number $(1)^2 - (1) + 11 = 11$ is prime.

2) With $n = 2$: the number $(2)^2 - (2) + 11 = 13$ is prime.

3) With $n = 3$: the number $(3)^2 - (3) + 11 = 17$ is prime.

three examples where $P(n)$ is true.

4) So the number $n^2 - n + 11$ is always prime.

End of proof

The only times when a universal statement can be proven using examples is when the domain is a finite set. (method of exhaustion)

Observe

When $n = 11$, the number is $(11)^2 - (11) + 11 = 11^2 = 11 \cdot 11$

not prime

[5] Here is a statement and a proposed proof. Comment on the validity or invalidity.

Statement S: The difference of the squares of any two consecutive integers is odd. True!

Proof

1) Let 6 and 7 be any two consecutive integers.
2) Then $7^2 - 6^2 = 49 - 36 = 13$ which is odd

End of proof

invalid proof
cannot prove a universal statement with infinite domain by example.

Proof needs to be a general proof.

See video for homework H04.2