Oct 22, 2020

Review for MATH 3050 Exam 3

Start with Section 8.2 Properties of Relations

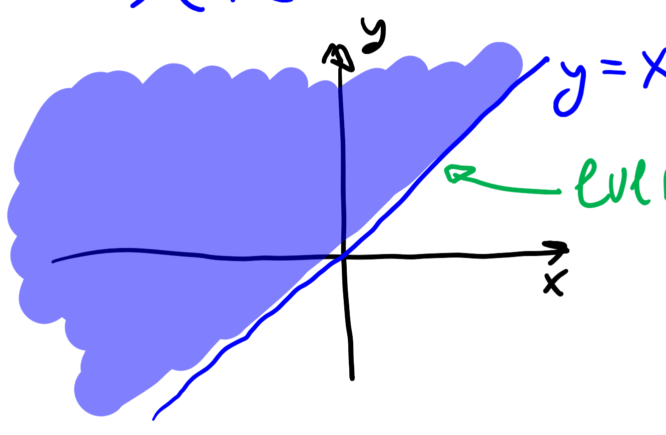
Example example R on the set $\mathbb{R}$

defined by xRy means $x \le y$

Reflexive?

Must show that for any x, xRx is true.

xRx means $x \le x$, which is always true.



every point on the main diagonal is part of the set.

Symmetric: for all x, y, If xRy then yRx

for all x, y, If $x \leq y$ then $y \leq x$

Not Symmetric:

$\exists x, y$ such that $x \leq y$ and $y \not\leq x$

Let $x = 2$, $y = 3$ Then $2 \leq 3$ is true

but $3 \leq 2$ is false

$2 \leq 3$ and $3 \not\leq 2$

Transitivity

for all x, y, z if $x \le y$ and $y \le z$ then $x \le z$,

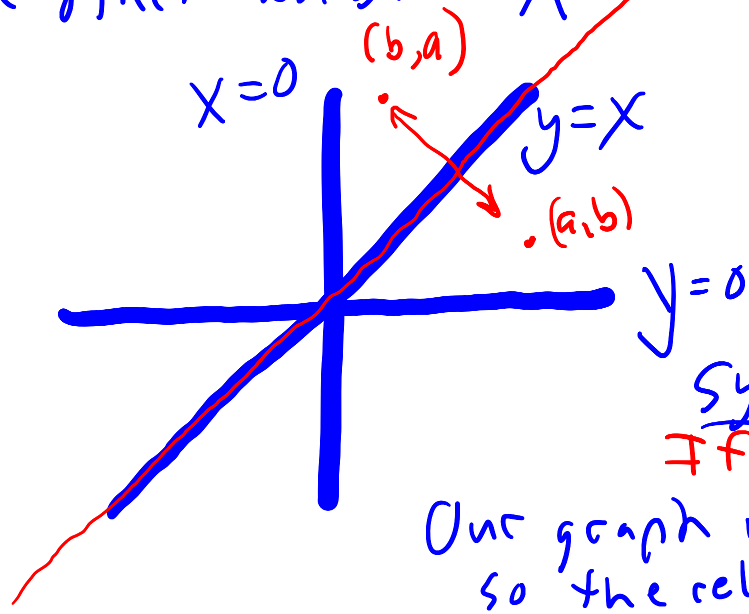
Property of Real Numbers

Example 2

Relation R on the set of real numbers $\mathbb{R}$

xRy means $(y-x) \cdot x \cdot y = 0$

In other words: $x=0$ or $y=0$ or $x=y$



Reflexive

for every x, $x=x$ is true

So xRx is true

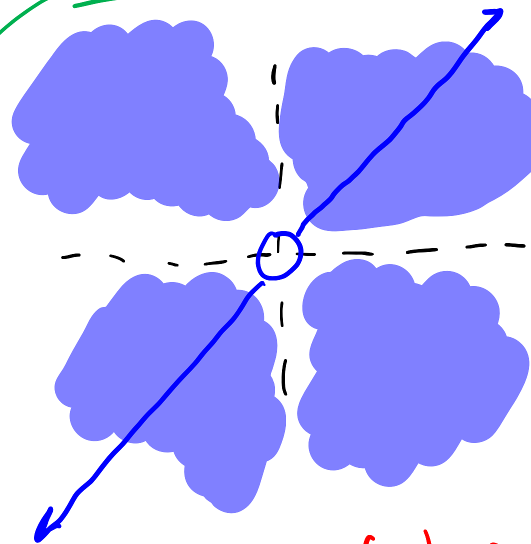
Graph contains every point of the form (x,x). The whole main diagonal.

Symmetric

If aRb then bRa

Our graph is symmetric across the line $y=x$

so the relation is symmetric



For Comparison: this is the graph of the relation xRy means $xy \neq 0$

missing one point on the main diagonal!

$0 \cancel{R} 0$ because $0 \cdot 0 \neq 0$ is false

$0 \cdot 0 = 0$

Not reflexive

Abstract Proof that R is Symmetric

Must prove $\forall x, y$ If xRy then yRx

(1) Suppose x, y are real numbers such that xRy

(2) $x = 0$ or $y = 0$ or $x = y$

(3) $y = 0$ or $x = 0$ or $y = x$

by 2 and commutative property of OR and symmetric property of real number equality

(4) Then yRx

End of Proof

Is R transitive?

Transitive: $\forall x,y,z$ (If xRy and yRz then xRz)

Not transitive: $\exists x,y,z$ (xRy and yRz and $x \not R z$)

Proof that R is not transitive

Let $x=1$, $y=0$, $z=2$

notice xRy is true (because $y=0$)

yRz is true (because $y=0$)

xRz is false (because neither x nor z is zero)

Functions $f: X \to Y$

Definition of one-to-one function (injective)

If $f(x_1) = f(x_2)$ then $x_1 = x_2$

If the two outputs are the same, then the inputs must have been the same.

contrapositive: If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

If two inputs are not the same then the resulting outputs are not the same

For every $y \in Y$ there is at most one $x \in X$ such that $y = f(x)$

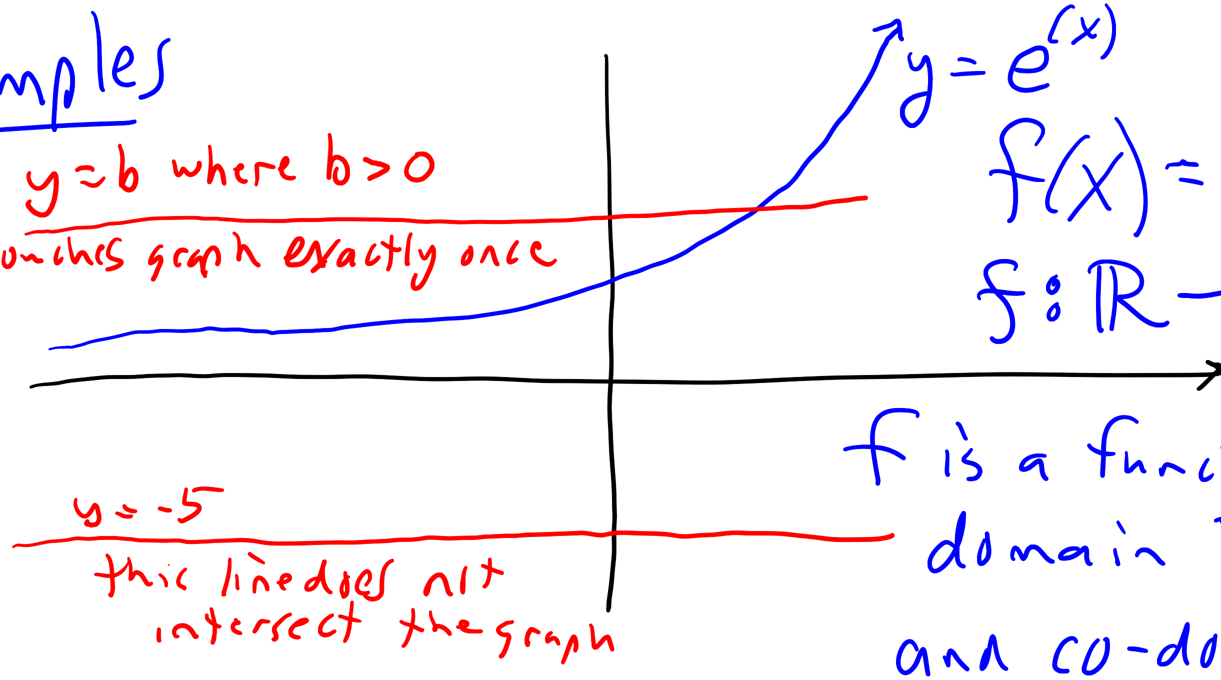
Definition of onto function (surjective)

For every $y \in Y$ there is an $x \in X$ such that $y = f(x)$

$$\forall y \in Y \, (\underbrace{\exists x \in X}_{\text{at least one } x} \, (y = f(x)))$$

For every $y \in Y$ there is at least one $x \in X$ such that $y = f(x)$

Examples



$f(x) = e^{(x)}$

$f : \mathbb{R} \to \mathbb{R}$

f is a function with domain $\mathbb{R}$ and co-domain $\mathbb{R}$

Not onto

Let $y = -5$. There is no $x \in \mathbb{R}$ such that $e^{(x)} = -5$

It is one-to-one

Definition of bijective (one-to-one correspondence)

both injective and surjective (one-to-one and onto)

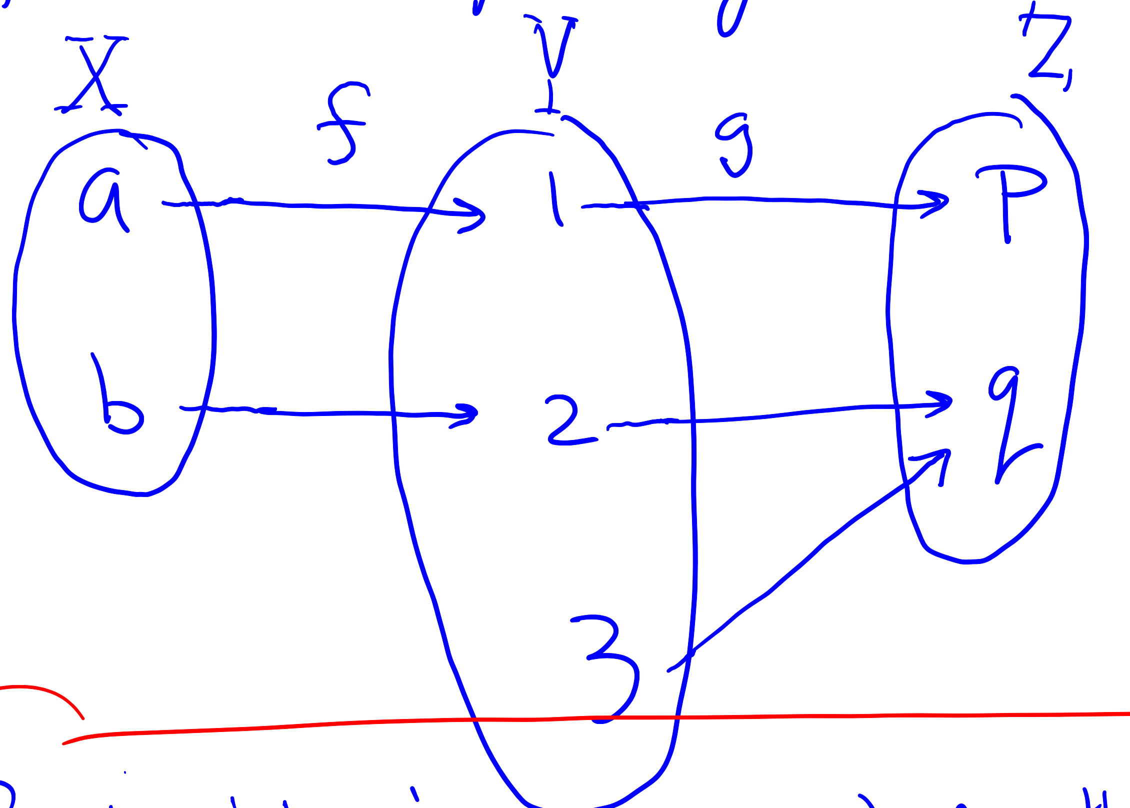
For every $y \in Y$ there is <u>exactly one</u> $x \in X$ such that $y = f(x)$

$$\forall y \in Y \left(\exists ! x \in X \left(y = f(x) \right) \right) \quad \text{bijective}$$

$$\forall y \in Y \left(\exists \text{ at most one } x \in X \left(y = f(x) \right) \right) \quad \text{injective}$$

$$\forall y \in Y \left(\underbrace{\exists x \in X}_{\text{at least one}} \left(y = f(x) \right) \right) \quad \text{surjective}$$

What is this diagram good for?



$f: X \to Y$

$g: Y \to Z$

$g \circ f: X \to Z$

$g \circ f$ is injective (one-to-one) even though g is <u>not</u> injective

$g \circ f$ is surjective (onto) even though f is <u>not</u> surjective

Theorems

If $g \circ f$ is injective then f must be injective

If $g \circ f$ is surjective then g must be surjective