Section 2.3: Valid and Invalid Arguments

Arguments and Argument Forms

Definition of Argument

Symbol:

$$\begin{array}{c} Premise_1 \\ Premise_2 \\ \vdots \\ Premise_k \\ \therefore Conclusion \end{array}$$

Usage: $Premise_1, Premise_2, \ldots, Premise_k, Conclusion$ are *statements*.

Meaning in symbols: $(Premise_1 \wedge Premise_2 \wedge \ldots \wedge Premise_k) \rightarrow Conclusion$

Meaning in words:

$$IF\ (Premise_1\ AND\ Premise_2\ AND\ \ldots\ AND\ Premise_k)\ THEN\ Conclusion$$

Remark: An *argument* is simply a different way of abbreviating and presenting a big *conditional statement.* So an *argument* is a *statement*.

[Example 1]

The capital of Ohio is Columbus

5 > 7

Kangaroos are marsupials

∴ The moon is made of green cheese

Simply means

IF ((The capital of Ohio is Columbus) AND (5 > 7) AND (Kangaroos are marsupials))

THEN The moon is made of green cheese.

Question: is the conditional statement represented by the above argument a true conditional statement?

IF F THEN F

True!

$F \rightarrow F$

Definition of Argument Form: Same symbol as argument, but, the premises and conclusion are all *statement forms*, not *statements*.

Definition of Argument form

Symbol:

$$
\begin{array}{r}
Premise_1 \\
Premise_2 \\
\vdots \\
Premise_k \\
\therefore Conclusion
\end{array}
$$

Usage: $Premise_1, Premise_2, \ldots, Premise_k, Conclusion$ are *statement forms*.

Meaning in symbols: $(Premise_1 \wedge Premise_2 \wedge \ldots \wedge Premise_k) \rightarrow Conclusion$

Meaning in words:

$$IF\ (Premise_1\ AND\ Premise_2\ AND\ \ldots\ AND\ Premise_k)\ THEN\ Conclusion$$

Remark: An *argument form* is simply a different way of abbreviating and presenting a big *conditional statement form.* So *an argument form* is a *statement form*.

[Example 2]

[Example]	**argument forms**	**corresponding statement forms**
[2a]	$p \to q$ p $\therefore q$	IF $((p \to q)$ AND $p)$ THEN q $((p \to q) \wedge p) \longrightarrow q$
[2b]	$p \to q$ $\sim q$ $\therefore \sim p.$	$((p \to q) \wedge \sim q) \longrightarrow \sim p$ IF $((p \to q)$ AND $\sim q)$ THEN $\sim p$
[2c]	$p \to q$ q $\therefore p$	$((p \to q) \wedge q) \longrightarrow p$ IF $((p \to q)$ AND $q)$ THEN p
[2d]	$p \to q$ $\sim p$ $\therefore \sim q$	$((p \to q) \wedge \sim p) \longrightarrow \sim q$

Valid and Invalid Argument Forms

Definition of a Valid Argument Form:

No matter what statements are substituted in for the statement variables in the premises, if the resulting premises are all true, then the conclusion is also true.

Definition of an Invalid Argument Form:

It is possible to substitute statements in for the statement variables in the premises in a way that the resulting premises are all true, and yet the conclusion is false.

Practically, how do we test an argument form to determine if it is valid? Make a truth table.

[Examples] Use truth tables to determine if the argument forms from the previous example are valid argument forms or invalid argument forms

[Example 2a]

$$p \to q$$ premise *
$$p$$ premise *
$$\therefore q$$

critical row

* p	q	* $p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

In all of the rows where all the premises are true, the conclusion is true. (critical rows)

Valid

[Example 2b]

$$p \to q$$
$$\sim q$$
$$\therefore \sim p.$$

p	q	~p	~q	p → q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

critical row

In every critical row, the conclusion is true

Valid Argument form.

[Example 2c]

$p \to q$ *

q *

$\therefore p$ *

p *	q *	$p \to q$ *
T	T	T
T	F	F
F	T	T
F	F	F

critical rows

There is a critical row with a false conclusion.

So invalid argument form.

[Example 2d]

$p \to q$ *

$\sim p$ *

$\therefore \sim q$ *

p	q	~p	~q	p → q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Critical rows

There is a critical row with false conclusion.

So the argument form is invalid

Famous Valid and Invalid Argument Forms: So far, we have seen two valid argument forms and two invalid argument forms. It turns out that they are famous, and are given names

[Example]	argument forms	Valid?	Name
[2a]	$p \to q$ p $\therefore q$	Valid	Modus Ponens
[2b]	$p \to q$ $\sim q$ $\therefore \sim p.$	Valid	Modus Tollens
[2c]	$p \to q$ q $\therefore p$	Invalid	Converse Error
[2d]	$p \to q$ $\sim p$ $\therefore \sim q$	Invalid	Inverse Error

.

Checking Validity of More Complicated Argument Forms

[Example 3] Use a truth table to determine if the argument form is valid:

$p \vee q$

$p \to \sim q$

$p \to r$

$\therefore r$

p	q	r	~q	p ∨ q	p → ~q	p → r
T	T	T	F	T	F	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	T	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

three critical rows

There is a row where the conclusion is false.
So invalid argument form.

Valid and Invalid Arguments

Definition of a Valid Argument:

An *argument* is called *valid* if its associated *argument form* is a *valid argument form*.

Definition of an Invalid Argument:

An *argument* is called *invalid* if its associated *argument form* is an *invalid argument form*.

[Example 4a] Consider this argument:

If $\overbrace{\text{Cleveland is in Athens county}}^{p}$, then $\overbrace{\text{Cleveland is in Ohio}}^{q}$

Cleveland is in Ohio

Therefore, Cleveland is in Athens County

(a) Use symbols to write the logical form of the argument.

(b) If the argument is valid, give the name of its form. If the argument is invalid, give the name of the error that is made.

$$\begin{aligned} &p \to q \\ &q \\ &\therefore p \end{aligned}$$

Invalid form. (Converse error)

Invalid Argument

Observations:

Hypotheses are <u>true</u>, but conclusion is false.

[Example 4b] Consider this argument:

If Nelsonville is in Athens county, then Nelsonville is in Ohio

Nelsonville is in Ohio

Therefore, Nelsonville is in Athens County

(a) Use symbols to write the logical form of the argument.

(b) If the argument is valid, give the name of its form. If the argument is invalid, give the name of the error that is made.

$$\left.\begin{array}{l} p \to q \\ q \\ \therefore p \end{array}\right\} \text{Invalid argument form (converse error again)}$$

So the argument is invalid

Observations:

Hypotheses are true and the conclusion is true.

[Example 4c] Consider this argument:

If Nelsonville is in Athens county, then Nelsonville is in Ohio p q

Nelsonville is in Athens County

Therefore, Nelsonville is in Ohio

(a) Use symbols to write the logical form of the argument.

(b) If the argument is valid, give the name of its form. If the argument is invalid, give the name of the error that is made.

$$\left.\begin{array}{l} p \to q \\ p \\ \therefore q \end{array}\right\} \text{valid argument form (modus ponens)}$$

Valid argument

Observations:

Valid argument with true hypotheses & true conclusion.

[Example 4d] Consider this argument: p q

If Cleveland is in Athens county, then Cleveland is in Ohio

Cleveland is in Athens County False!

Therefore, Cleveland is in Ohio

(a) Use symbols to write the logical form of the argument.

(b) If the argument is valid, give the name of its form. If the argument is invalid, give the name of the error that is made.

$$\begin{array}{l} p \to q \\ p \\ \therefore q \end{array}$$

valid argument form
Modus ponens

Valid argument.

Observations:

Valid argument with false hypothesis and true conclusion.

[Example 4e] Consider this argument: p q

If Pittsburgh is in Athens county, then Pittsburgh is in Ohio

Pittsburgh is in Athens County false

Therefore, Pittsburgh is in Ohio false

(a) Use symbols to write the logical form of the argument.

(b) If the argument is valid, give the name of its form. If the argument is invalid, give the name of the error that is made.

$p \to q$

p

$\therefore q$

Valid argument form (modus ponens)

Valid argument.

Observations:

Valid argument with false hypothesis and false conclusion.

Observations about Invalid and Valid Arguments

It is possible to have an invalid argument with *true hypotheses* and a *true conclusion*. **[Example 4b]** was this type.

It is possible to have a valid argument with false hypotheses and a true conclusion. **[Example 4d]** was this type.

It is possible to have a valid argument with false hypotheses and a false conclusion. **[Example 4e]** was this type.

So surprising things can happen in arguments, regardless of whether they are called valid arguments or invalid arguments.

Sound Argument

Of course, the weird things that can happen in arguments are all things that we want to *avoid*. It is helpful to be able to recognize the weird things in order to better avoid them.

But we are most interested in learning how to build valid arguments with *true hypotheses*. Note that if an argument is valid and has true hypotheses, then it will automatically have a *true conclusion*. **[Example 4c]** was this type. We give those kinds of arguments a special name.

> An *argument* is called ***sound*** if it is *valid* and has *true hypotheses*.

With that terminology, we could state our goal of the first part of this course as

> Our goal is to learn how to build *sound arguments*.

Rules of Inference

So far, we have only encountered two valid argument forms: *modus ponens* and *modus tollens*. They get special names, because they occur very frequently.

There are other valid argument forms that also occur frequently and are also given names. They are presented in Table 2.3.1 of the book. Collectively they are called *Rules of Inference*.

TABLE 2.3.1 Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to \mathbf{c}$ $\therefore p$

Using Rules of Inference to Build and Justify Valid Argument Forms

Two pages ago, our goal was stated: *to learn how to build sound arguments*.

That means that we want to build arguments that are *valid* and have *true hypotheses*.

And that means that we want to build arguments that have *valid argument forms* and have *true hypotheses*.

We focus now on the first part of that goal: *learning how to build valid argument forms*.

It turns out that a good way to learn how to build valid argument forms is to build them up from other known valid argument forms. At each step of the building, one states which known valid argument form is being used. This process is called ***deducing the conclusion from the premises***.

Method of using the Rules of Inference to Deduce a Conclusion from Given Premises

Preparation: Start by writing down the premises, each on a line with

- a number in front
- the premise
- the word (premise) in parentheses afterwords

Draw a dotted line after the last premise.

Add a numbered line, containing

- a number in front
- a conclusion that makes a new valid argument form,
- a justification in the form of a citation of a *Rule of Inference*
  (with numbers in the citation referring to previous steps that were used)

Continue adding numbered lines until the desired conclusion is reached.

That desired conclusion will be the last line. Draw a dotted line above the last line.

Knowing how to use the Rules of Inference in this manner is an acquired skill. One must have in mind what steps are needed, and how those steps could be justified using known valid argument forms. (the rules of inference)

One way to start to understand the process is to take a *given* argument form that is *known* to be valid, and to fill in intermediate conclusions with justifications (provided by the *Rules of Inference*) that will make it clear why the given argument form is valid.

That is what we will do in our last example of the video. (You have a similar exercise in the homework.)

[Example 5] Deducing a conclusion from premises

The argument form shown at right is a *valid argument form*.

The goal is to verify that it is a valid argument form. But verifying the validity using a truth table would be brutal, because there are 6 statement variables. The truth table would need $2^6 = 64$ rows! That work would not make you any smarter, and nobody would want to read it.

$\sim p \to r \wedge \sim s$

$t \to s$

$u \to \sim p$

$\sim w$

$u \vee w$

$\therefore r$

Instead, a far smarter way to verify the validity would be to do the following:

Use the *Rules of Inference* to *deduce the conclusion from the premises*.

(Number each line and give clear justifications with references.)

Solution

1. $\sim p \to r \wedge \sim s$ premise
2. $t \to s$ premise
3. $u \to \sim p$ premise
4. $\sim w$ premise
5. $u \vee w$ premise

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6. $\therefore u$ by Elimination(a) on 5,4
7. $\therefore \sim p$ by Modus ponens on 3,6
8. $\therefore r \wedge \sim s$ by Modus ponens on 1,7

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9. $\therefore r$ by specialization(a) on 8.

End of [Example 5]

End of Video