																		,											
L	A	S	T		N	A	M	E										,	F	I	R	S	T		N	A	M	E	

Homework H03

MATH 3110/5110 (Barsamian)

Due at start of class Fri Jan 28, 2022

Problem:	1	2	3	4	5	6	Total	Rescaled
Your Score:								
Possible:	20	20	20	20	20	20	120	10

There is a large collection of ***Suggested Exercises***. (These exercises are not to be turned in and are not graded, but you should write solutions for as many of them as possible and keep your solutions in a notebook for study.)

2.1 # 1, 3, 5, 6, 8, 10, 11, 12, 13, 16, 18, 19, 24, 25

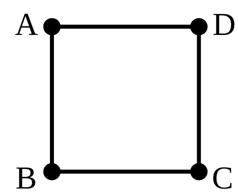
2.2 # 1, 2, 4, 5, 6, 7ii, 9, 10, 11, 12, 17, 18i, 19, 20

2.3 # 1, 2, 3, 4, 5, 6

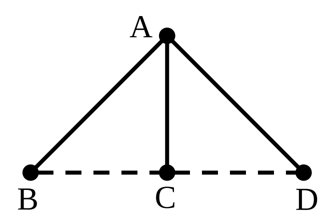
[1] Finite Geometries (Concepts from Section 2.1, Videos 2.1a,b)

For each pair $(\mathcal{P}, \mathcal{L})$, is the pair qualified to be called an *abstract geometry*? An *incidence geometry*? Explain.

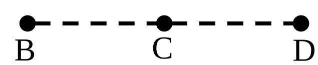
(a) Pair $(\mathcal{P}, \mathcal{L})$ consisting of
- points $\mathcal{P} = \{A, B, C, D\}$
- lines $\mathcal{L} = \{\{A, B\}, \{B, C\}, \{C, D\}, \{D, A\}\}$



(a) Pair $(\mathcal{P}, \mathcal{L})$ consisting of
- $\mathcal{P} = \{A, B, C, D\}$
- $\mathcal{L} = \{\{A, B\}, \{A, C\}, \{A, D\}, \{B, C, D\}\}$ (The dotted thing is one line.)



(a) Pair $(\mathcal{P}, \mathcal{L})$ consisting of
- points $\mathcal{P} = \{B, C, D\}$
- lines $\mathcal{L} = \{\{B, C, D\}\}$ (The dotted thing is one line.)



[2] Finding Equations for Cartesian Lines and Poincaré Lines (Concepts from Section 2.1, Video 2.1a)

(a) The Cartesian line through $P = (2,1)$ and $Q = (4,3)$.

(b) The Poincaré line through $P = (2,1)$ and $Q = (4,3)$.

- Find the equation for the lines. Present your results with a line symbol and set notation.
- Graph the lines on the same set of axes.
- Make your graph large (at least 4" × 4") and neat, and label and put (x, y) coordinates on all important locations: points P, Q, all axis intercepts (for the Cartesian line), the center of the circle and the missing endpoints (for the Poincaré line)

Hint: See *Procedures for Finding Cartesian and Poincaré Lines* and **[Example 2]** in the Notes for Video 2.1a.

The Homework continues on back ➔

[3] Collinearity in the Cartesian plane and the Poincaré plane. (Concepts from Section 2.1, Videos 2.1a,b)

For each question, find examples of points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ and $R = (x_3, y_3)$ satisfying the stated requirements. Draw an illustration for each part **(a)**, **(b)**, **(c)**, **(d)**, showing the points clearly labeled with their coordinates and any lines that exist clearly labeled with their description. If you claim that P, Q, R are collinear, then you must give the line equation for the line that they all lie on. If you claim that P, Q, R are non-collinear, then you must explain how you know that they are non-collinear.

(a) P, Q, R are collinear in both the Cartesian plane and the Poincaré plane.

(b) P, Q, R are collinear in the Cartesian plane and non-collinear in the Poincaré plane.

(c) P, Q, R are non-collinear in the Cartesian plane and collinear in the Poincaré plane.

(d) P, Q, R are non-collinear in both the Cartesian plane and the Poincaré plane.

Recall that there are two common questions about geometries. I call them the **BIG QUESTIONS**.

- **BIG QUESTION #1:** *Do parallel lines exist?*
- **BIG QUESTION #2:** *Given a line L and a point P not on L, how many lines exist that contain P and are parallel to L?*

[4] (Concepts from Section 2.1, Video 2.1a)

Examples about BIG QUESTION #2 in the Cartesian Plane, the Poincaré plane, and the Riemann sphere.

(a) Describe all lines through $(0,1)$ that are parallel to the vertical line L_5 in the Cartesian plane

(b) Describe all lines through $(0,1)$ that are parallel to the type I line ${}_5L$ in the Poincaré plane

(c) Describe all lines through the north pole $N = (0,0,1)$ that are parallel to the equator in the Riemann sphere.

[5] (Concepts from Section 2.1, Videos 2.1a,b) Theorem 2.1.6 is the first theorem in our book about the *behavior* of *points* and *lines* in *abstract geometry*. The theorem and its **Corollary 2.1.7** will be used throughout our course. The book provides a proof of Theorem 2.1.6 on page 24. As with all of the book's proofs, the proof is presented in *paragraph form*, with no justifications. Rewrite it in *two-column form*, with each line consisting of a numbered statement and a justification.

[6] (Concepts from Section 2.2, Video 2.2a) Distance in various *metric geometries*.

For each part, give an exact, simplified answer and a decimal approximation, rounded to 3 decimals.

(a) Find the *Euclidean distance* between $P = (2,1)$ and $Q = (4,3)$. That is, find $d_E(P, Q)$.

(b) Find the *Taxicab distance* between $P = (2,1)$ and $Q = (4,3)$. That is, find $d_T(P, Q)$.

(c) Find the *Poincaré distance* between $P = (2,1)$ and $Q = (4,3)$. That is, find $d_{\mathbb{H}}(P, Q)$.

.