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Homework H04
MATH 3110/5110
Due Mon, Feb 14, 2022

Problem:	1	2	3	4	5	6	7	8	Total	Rescaled
Your Score:										
Possible:	20	20	20	20	20	20	20	20	160	10

This Homework is worth extra points because it contains eight problems instead of the usual five.

Suggested Exercises: 3.1#1, 5, 7, 8, 9b
3.2 # 1, 2, 3, 5, 7, 8, 9, 10, 13

Assigned Exercises:

[1] Prove Proposition 3.1.1 parts (iii), (vi), (viii). For each part, the clarity of the presentation is important.

- Use Coordinates $A = (x_A, y_A)$ and $B = (x_B, y_B)$.
- Show the calculation of the *left side* of the equality.
- Show the calculation of the *right side* of the equality.
- Confirm that *left side* = *right side*.

[2] (See Section 3.1 and Video 3.1 for reference)

(a) Let $A = (3,2)$ and $B = (6,4)$ and $C = (3,4)$ and $D = (-2,3)$ and $E = (-3,-2)$

Do the following:

Illustrate the vectors A, B, C, D, E by plotting them all on the same set of axes. (Just one graph for all five vectors.) Make your illustration large (at least 4" × 4") and neat. Put the tails of the vectors at $(0,0)$, and put (x, y) coordinates on the heads of the vectors.

For **each** of these four pairs: A, B and A, C and A, D, and A, E

(i) Confirm that the pair sastisfies the *Cauchy-Schwarz Inequality* $|\langle P, Q\rangle| \leq \|P\| \cdot \|Q\|$.

(ii) Confirm the pair satisfies the *triangle inequality for the norm* $\|P + Q\| \leq \|P\| + \|Q\|$.

In (i) and (ii), the clarity of the presentation is important.

- Show the calculation of the *left side* of the inequality.
- Show the calculation of the *right side* of the inequality.
- Confirm that *left side* ≤ *right side*.

The Homework continues on back ➔

[3] (See Section 3.1 and Video 3.1) Let $P = (2,3)$ and $Q = (-1,7)$ and $R = (8,-5)$

(a) Use vector notation to build a description of line $\overleftrightarrow{PQ}$. Show how the *initial presentation* involving *vector calculations* can be *simplified* to a *final presentation* that is a *parametric form* of the description of the line.

(b) Use vector notation to build a ***special ruler with P as origin and Q positive.***

(c) Notice that point R lies on line $\overleftrightarrow{PQ}$. Using the *special ruler* from part (b), find the coordinate of point R. (Show clearly how the calculation works.)

[4] Let $A = (3,5)$, $B = (15,13)$, and $D(20,12)$ in the *Poincaré Plane*. Show that $A - B - C$.

[5] (3.2#10) In the Taxicab Plane, find three points A, B, C which are not collinear but satisfy the equation

$$d_T(A,C) = d_T(A,B) + d_T(B,C).$$

(This problem shows why the definition of *betweenness* includes the requirement that the points be *collinear*.)

[6] Prove: If P, Q, R are collinear points in a metric geometry and there exists a ruler f for line $\overleftrightarrow{PQ}$ with the property that $f(P) < f(Q) < f(R)$, then $P - Q - R$.

[7] (3.2 concepts) Justifying and Illustrating the Steps in a Given Proof.

In Video 3.2b, I provide a proof of Theorem 3.2.6. Rewrite the proof, keeping my text but providing **four justifications** and **five illustrations** where indicated. The justifications should reference earlier statements in the proof or definitions, theorems, propositions, or exercises in the book. The illustrations should be large and neat.

[8] (2.1 concepts)

(a) Prove *The One Line Theorem of Abstract Geometry*:

Given any point P in an *abstract geometry*, there exists *at least one line* that contains P.

(b) Prove *The Two Line Theorem of Incidence Geometry*:

Given any point P in an *incidence geometry*, there exist *at least two distinct lines* that contain P.