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Homework H06
MATH 3110/5110
Due Mon, Feb 28, 2022

Problem:	**1**	**2**	**3**	**4**	**5**	**Total**	**Rescaled**
Your Score:							
Possible:	**20**	**20**	**20**	**20**	**20**	**100**	**10**

Suggested Exercises: 4.1 # 1, 2, 4, 5, 6, 8, 9, 10, 11, 13
4.2 # 1, 4
4.3 # 1, 2, 3, 4, 7

Assigned Exercises:

[1] Prove that in a metric geometry, a triangle ΔABC is *not* a convex set.

[2] (4.1#10) Prove Theorem 4.1.4:

Given: a line l and points P, Q, R in a metric geometry which satisfies PSA.

Claim: If P, Q are on opposite sides of l and Q and R are on *opposite sides* of l, then P and R are on the *same side* of l.

[3] (Based on Exercise 4.1#5) Justify and illustrate the steps in the given proof of this statement:

If L is a line in a metric geometry that satisfies the Plane Separation Axiom, then each of the half-planes determined by L contains three non-collinear points.

Proof

(1) Suppose L is a line in a metric geometry that satisfies the Plane Separation Axiom, and suppose that H is one of the half-planes determined by L. **(Illustrate)**

(2) There exist two distinct points on L. Call them A and B. **(Justify) (Illustrate)**

(3) There exists a point C in H. (By the result of 4.1#4, which was discussed in Video 4.1) **(Illustrate)**

(4) There exists a unique line passing through A and C. **(Justify)**

(5) The line passing through A and C is not L. **(Justify)** So it must be new. Call it M. **(Illustrate)**

(6) There exists a unique line passing through B and C. **(Justify)**

(7) The line passing through B and C is not L or M. **(Justify)** So it must be new. Call it N. **(Illustrate)**

(8) There exists a point such that $A - C - point$. **(Justify)**

(9) This point cannot be the same as any of our previous three points. **(Justify)** So it must be a new point. Call it D. So $A - C - D$. **(Illustrate)**

(10) Point D is in half-plane H. **(Justify) (Illustrate)**

(11) There exists a point such that $B - C - point$. **(Justify)**

(12) This point cannot be the same as any of our previous four points. **(Justify)** So it must be a new point. Call it E. So $B - C - E$. **(Illustrate)**

(13) Point E is in half-plane H. **(Justify) (Illustrate)**

(14) Points C, D, E are non-collinear. **(Justify)** And notice that all three lie in half-plane H. **(Illustrate)**

(15) We have not said which of the two half-planes determined by L we are talking about. So all that we have said holds for both half-planes. That is, each of the half-planes contains three non-collinear points.

End of Proof.

[4] (Based on 4.3#2) Justify and illustrate the steps in the given proof of this statement:
In a metric geometry that satisfies PSA, if ΔABC and points D, F have properties $B-C-D$ and $A-F-B$, then there exits a point E such that $D-E-F$ and $A-E-C$.

Proof

(1) Suppose Metric Geometry $\mathcal{M} = (\mathcal{P}, \mathcal{L}, d)$ satisfies the Plane Separation Axiom and that triangle ΔABC and points D, F have the property that $B-C-D$ and $A-F-B$. **(Illustrate.)**

Part 1: Show that there exists a point $\boldsymbol{E}$ on line $\overleftrightarrow{\boldsymbol{AC}}$ such that $\boldsymbol{D-E-F}$

(2) Line $\overleftrightarrow{AC}$ creates two half planes. **(Justify.) (Illustrate.)**

(3) Points B, F are in the same half plane of line $\overleftrightarrow{AC}$. **(Justify.) (Illustrate.)**

(4) Points B, D are in different half planes of line $\overleftrightarrow{AC}$. **(Justify.) (Illustrate.)**

(5) Therefore, points F, D are in different half planes of line $\overleftrightarrow{AC}$. **(Justify.) (Illustrate.)**

(6) Line $\overleftrightarrow{AC}$ intersects segment $\overline{DF}$ at a point such that $D - point - F$. **(Justify.)**

(7) $Point$ cannot be any of our previous points: ($Point$ cannot be A. **(Justify.)** $Point$ cannot be B. **(Justify.)** $Point$ cannot be C. **(Justify.)** $Point$ cannot be D or F. **(Justify.)**) Therefore, $point$ must be a new point. Call it E. So E is a point on line $\overleftrightarrow{AC}$ and $D-E-F$. **(Illustrate.)**

Part 2: Show that $\boldsymbol{A-E-C}$

(8) Line $\overleftrightarrow{BD}$ creates two half planes. **(Justify.) (Illustrate.)**

(9) Points A, F are in the same half plane of line $\overleftrightarrow{BD}$. **(Justify.) (Illustrate.)**

(10) Points F, E are in the same half plane of line $\overleftrightarrow{BD}$. **(Justify.) (Illustrate.)**

(11) Therefore, points A, E are in the same half plane of line $\overleftrightarrow{BD}$. **(Justify.) (Illustrate.)**

(12) Therefore, it must be true that $A-E-C$. **(Justify.) (Illustrate.)**

Conclusion

(13) We has shown that there is a point E such that $D-E-F$ and $A-E-C$. **(Illustrate.)**

End of Proof

[5] Provide an example showing that the *Missing Strip Plane metric geometry* does not satisfy *Peano's Axiom*.