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Homework H07
MATH 3110/5110
Due Friday, March 18, 2022

Problem:	1	2	3	4	5	6	7	Total	Rescaled
Your Score:									
Possible:	20	20	20	20	20	20	20	100	10

Notice that there are actually 14 points possible. Any points scored over 10 will be extra credit.

Suggested Exercises: 4.4 # 2,3,4,5,6,9,10,11,12,13,15,17,19,23,25

Assigned Exercises:

[1] (based on 4.4#4) Thm 4.4.6 says: Given $\angle ABC$ in a Pasch geometry, if $A-P-C$, then $P \in \text{int}(\angle ABC)$.

(a) Illustrate the statement of the Theorem.

(b) Justify and illustrate the steps in this proof of the Theorem.

(1) Suppose that in a Pasch geometry, $\angle ABC$ and point P satisfy $A-P-C$. **(Illustrate.)**

(2) Segment $\overline{PC}$ does not intersect line $\overleftrightarrow{BA}$. **(Justify.) (Illustrate.)**

(3) Points P, C are in the same half plane of line $\overleftrightarrow{BA}$. **(Justify.)** So $P \in H_{\overleftrightarrow{BA},C}$. **(Illustrate.)**

(4) Segment $\overline{PA}$ does not intersect line $\overleftrightarrow{BC}$. **(Justify.) (Illustrate.)**

(5) Points P, A are in the same half plane of line $\overleftrightarrow{BC}$. **(Justify.)** So $P \in H_{\overleftrightarrow{BC},A}$. **(Illustrate.)**

(6) So $P \in H_{\overleftrightarrow{BA},C} \cap H_{\overleftrightarrow{BC},A}$. **(Justify.)**

(7) Therefore, $P \in \text{int}(\angle ABC)$. **(Justify.) (Illustrate.)**

End of Proof

[2] (4.4#5) Here is a true Statement: In a Pasch geometry, if $P \in \text{int}(\angle ABC)$. then $\text{int}\left(\overrightarrow{BP}\right) \subset \text{int}(\angle ABC)$.

(a) Illustrate the Statement.

(b) Prove the statement: Provide clear illustrations for all the steps of your proof.

[3] (4.4#9) Theorem 4.4.8 says:

In a Pasch geometry, if $\overline{CP} \cap \overleftrightarrow{AB} = \phi$, *then* $P \in \text{int}(\angle ABC)$ *if and only if* A, C *are on opposite sides of* $\overleftrightarrow{BP}$.

As always, I prefer to state *If-and-Only-If* theorems using the terminology of *Equivalent Statements*.

In a Pasch geometry, given points A, B, C, D *with* $\overline{CP} \cap \overleftrightarrow{AB} = \phi$, *the following are equivalent (TFAE):*

(i) $P \in \text{int}(\angle ABC)$

(ii) A, C *are on opposite sides of* $\overleftrightarrow{BP}$.

Illustrate the statement of the Theorem. (**Don't Prove the Theorem!** Just illustrate the statement.)

[4](4.4#11) Here is a true Statement:

In a Pasch geometry, if $\overline{CP} \cap \overleftrightarrow{AB} = \phi$,

then either $\overrightarrow{BC} = \overrightarrow{BP}$ or $P \in \text{int}(\angle ABC)$ or $C \in \text{int}(\angle ABP)$. (exclusive or)

Illustrate the Statement. (**Don't Prove the Statement!** Just illustrate the statement.)

As discussed throughout the semester, a *conditional statement* is a statement of the form: *If A then B*
A conditional statement is *logically equivalent* to its *contrapositive*: *If NOT(B) then NOT(A).*
As a result of this, any time one proves a theorem that has the form of a conditional statement, one knows that the contrapositive version of the same statement is automatically true. The contrapositive statement is not another theorem: it is just a different way of saying the theorem that has already been proven.

But the original statement is *not* logically equivalent to its *converse*: *If B then A.* As a result of this, when a known theorem has the form of a conditional statement, the converse statement is not automatically true. (The converse statement is *not* just a different way of saying the theorem that has already been proven.) If the converse statement *is* true, then it constitutes *another theorem*, and it will have to be proven with a *new proof*.

That is the situation with the *Crossbar Theorem* and the *Converse of the Statement of the Crossbar Theorem*.

Crossbar Theorem (Proven in the book and in Video 4.4)(Restated a little differently here.)

Given $\angle ABC$ and point P in a Pasch geometry, if $P \in \text{int}(\angle ABC)$ then $\overrightarrow{BP}$ intersects $\text{int}(\overline{AC})$.

Converse of the Statement of the Crossbar Theorem (You will prove this in problem [5])

Given $\angle ABC$ and point P in a Pasch geometry, if $\overrightarrow{BP}$ intersects $\text{int}(\overline{AC})$, then $P \in \text{int}(\angle ABC)$.

[5](4.4#12) Prove the following Theorem (Converse of the Statement of the Crossbar Theorem)

Given $\angle ABC$ and point P in a Pasch geometry, if $\overrightarrow{BP}$ intersects $\text{int}(\overline{AC})$, then $P \in \text{int}(\angle ABC)$.

Provide clear illustrations for all the steps of your proof that can be visualized.

[6](a) Write the *contrapositive* of the statement of the *Crossbar Theorem*.
(b) Write the *contrapositive* of the *Converse of the Statement of the Crossbar Theorem*.

[7] (**Remark:** In Homework H06 problem [1], you proved that a triangle is *not* a convex set.)
(4.4#15) Prove Theorem 4.4.10: In a Pasch geometry, $\text{int}(\Delta ABC)$ *is* convex.