

### **3.4: Angles and Triangles**

**produced by Mark Barsamian, 2021.02.25**

**for Ohio University MATH 3110/5110 College Geometry**

#### **Topics**

- **Definitions of Angle and Triangle**
- **Example**
- **Extreme Points of Angles and Triangles**

**Reading:** Section 3.4 Angles and Triangle, p 59 - 62 in *Geometry: A Metric Approach with Models, Second Edition* by Millman & Parker

**Homework:** Section 3.4 # 1, 2, 3, 4

## Definitions of Angle and Triangle

### Definition of Angle

**Symbol:**  $\angle ABC$

**Spoken:** *angle A B C.*

**Usage:**  $A, B, C$  are noncollinear points in a metric geometry  $\mathcal{M} = (\mathcal{P}, \mathcal{L}, d)$ .

**Meaning:** the set  $\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$

**Additional Terminology:** The **vertex** of  $\angle ABC$  is the point  $B$ .

### Definition of Triangle

**Symbol:**  $\Delta ABC$

**Spoken:** *triangle A B C.*

**Usage:**  $A, B, C$  are noncollinear points in a metric geometry  $\mathcal{M} = (\mathcal{P}, \mathcal{L}, d)$ .

**Meaning:** the set  $\Delta ABC = \overline{AB} \cup \overline{BC} \cup \overline{CA}$

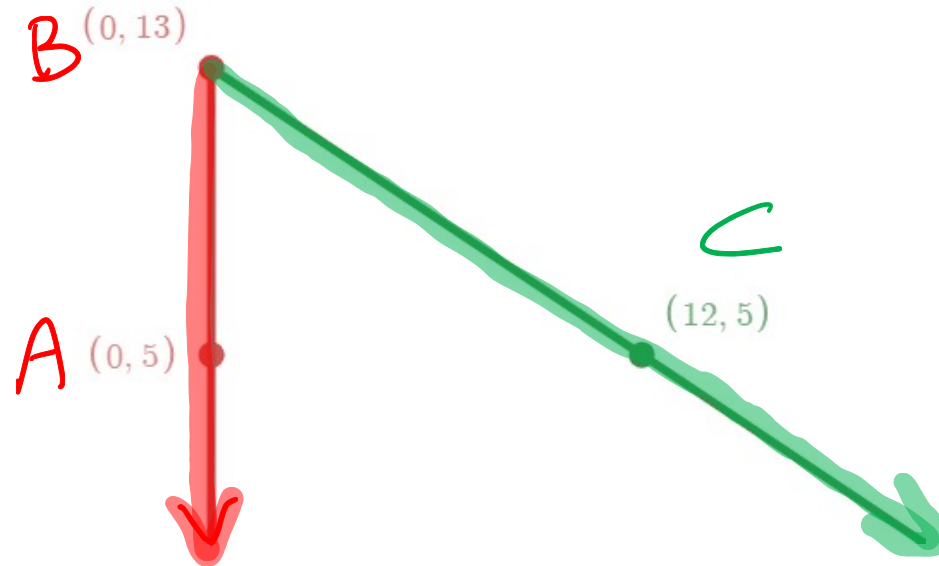
**Additional Terminology:**

The **vertices** of  $\Delta ABC$  are the points  $A, B, C$ .

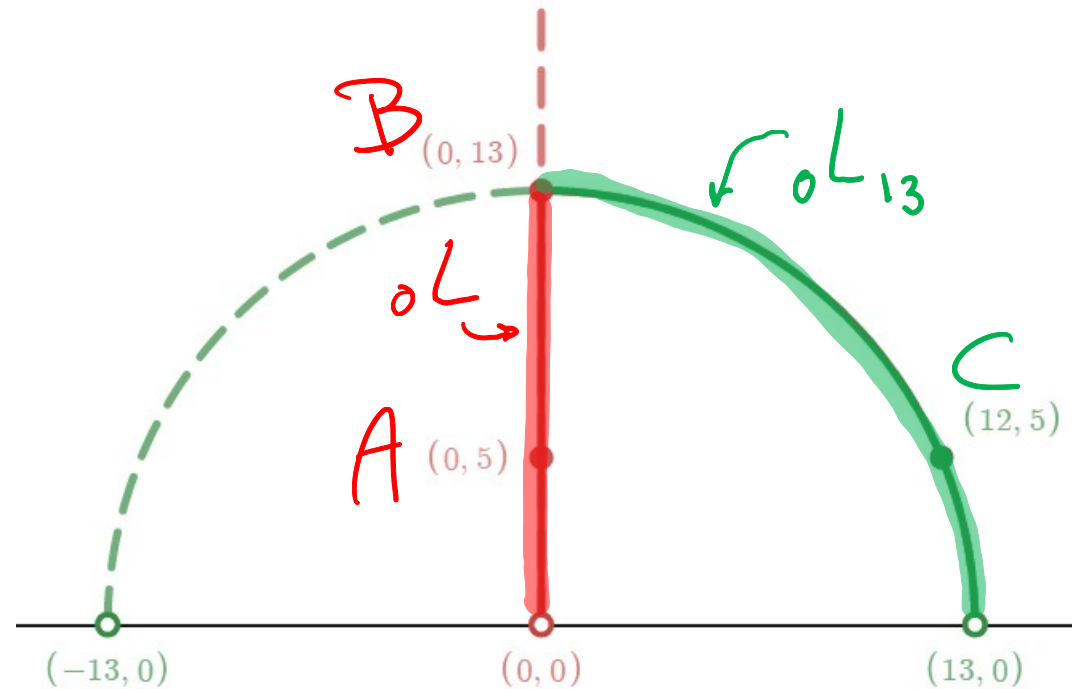
The **sides** (or **edges**) of  $\Delta ABC$  are the segments  $\overline{AB}, \overline{BC}, \overline{CA}$ .

[Example 1] Let  $A = (0,5)$  and  $B = (0,13)$  and  $C = (12,5)$

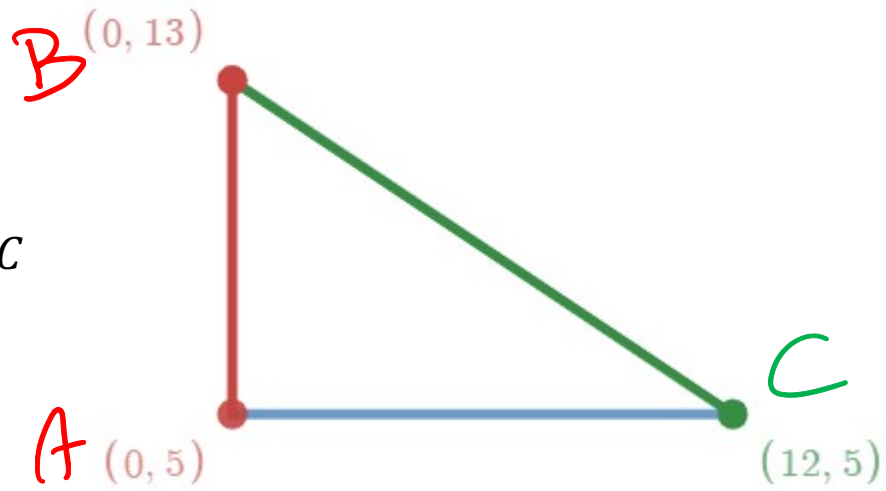
Euclidean angle  $\angle ABC$



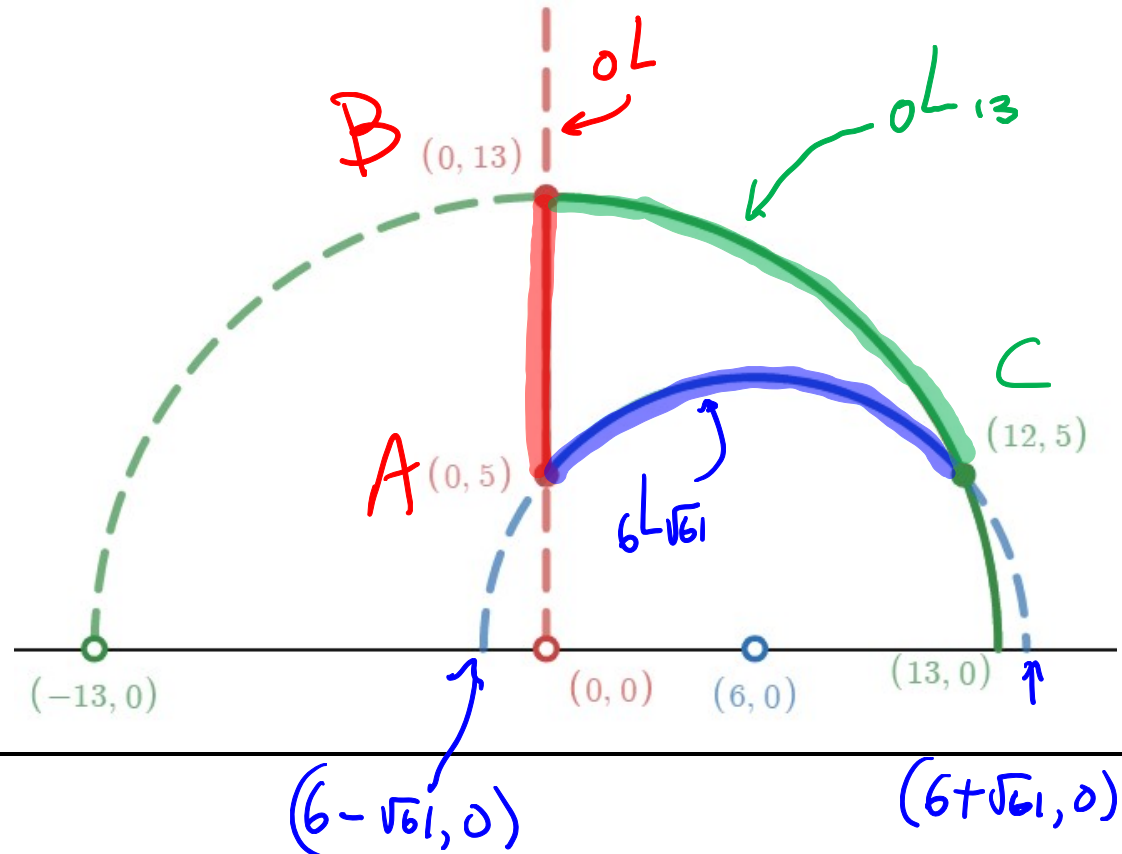
Poincaré angle  $\angle ABC$



Euclidean triangle  $\Delta ABC$



Poincaré triangle  $\Delta ABC$



End of [Example 1]

## Extreme Points of Angles and Triangles

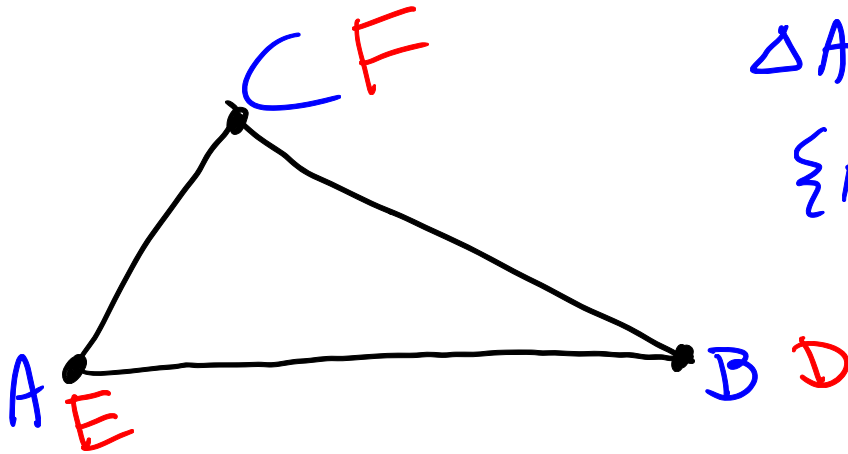
**Lemma 3.4.1:** The Only Extreme Point of an Angle is the Vertex

**Theorem 3.4.2 (Corollary)** If  $\angle ABC = \angle DEF$ , then  $B = E$ .

**Lemma 3.4.3:** The Only Extreme Points of ~~an Angle~~ <sup>a triangle</sup> are the vertices.

**Theorem 3.4.2 (Corollary)** If  $\triangle ABC = \triangle DEF$ , then  $\{A, B, C\} = \{D, E, F\}$ .

End of Video



$$\triangle ABC = \triangle DEF$$
$$\{A, B, C\} = \{D, E, F\}$$