Group Work GW03: Function Values and Limits for a Rational Function

Without using a calculator, answer the following questions about the functions.

$$f(x) = \frac{x^2 + 10x + 21}{x^2 = 5x + 6} = \frac{(x + 7)(x + 3)}{(x + 2)(x + 3)} \quad and \quad g(x) = \frac{x + 7}{x + 2}$$

Part 1: Function Values

(1) Find $f(-2)$ by substituting $x = -2$ into the factored version of f.

(2) Find $f(-3)$ by substituting $x = -3$ into the factored version of f.

(3) Find $f(-7)$ by substituting $x = -7$ into the factored version of f.

(4) Find $g(-2)$ by substituting $x = -2$ into the factored version of g.

(5) Find $g(-3)$ by substituting $x = -3$ into the factored version of g.

(6) Find $g(-7)$ by substituting $x = -7$ into the factored version of g.

(7) Is $\frac{(x + 7)(x + 3)}{(x + 2)(x + 3)} = \frac{x + 7}{x + 2}$ a true equation?

(8) Are $f(x)$ and $g(x)$ the same function? Explain.

Part 2: Limits

(9) Estimate $\lim_{x \to -3} f(x)$ using Section 1.1 Techniques. That is, compute these values:

$$f(-2.9) = \frac{\big((-2.9) + 7\big)\big((-2.9) + 3\big)}{\big((-2.9) + 2\big)\big((-2.9) + 3\big)} =$$

$$f(-2.99) = \frac{\big((-2.99) + 7\big)\big((-2.99) + 3\big)}{\big((-2.99) + 2\big)\big((-2.99) + 3\big)} =$$

$$f(-2.999) = \frac{\big((-2.999) + 7\big)\big((-2.999) + 3\big)}{\big((-2.999) + 2\big)\big((-2.999) + 3\big)} =$$

Estimate: $\lim_{x \to -3^+} f(x) =$

$$f(-3.1) = \frac{\big((-3.1) + 7\big)\big((-3.1) + 3\big)}{\big((-3.1) + 2\big)\big((-3.1) + 3\big)} =$$

$$f(-3.01) = \frac{\big((-3.01) + 7\big)\big((-3.01) + 3\big)}{\big((-3.01) + 2\big)\big((-3.01) + 3\big)} =$$

$$f(-3.001) = \frac{\big((-3.001) + 7\big)\big((-3.001) + 3\big)}{\big((-3.001) + 2\big)\big((-3.001) + 3\big)} =$$

Estimate: $\lim_{x \to -3^-} f(x) =$

Estimate: $\lim_{x \to -3} f(x) =$

(5) Factor f. (Check your factorizations by multiplying.)

(6) Are you allowed to cancel factors in the factored form of f? Explain why you think you are allowed to cancel, or why you are not.

(7) Find $f(1)$ by substituting $x = 1$ into the factored version of f.

(8) Find $f(2)$ by substituting $x = 2$ into the factored version of f.

(9) Find $f(3)$ by substituting $x = 3$ into the factored version of f.

Part 2: Limit as $x \to 3$

Using the factored form of f, compute the following values and guess the limits. **(No calculators)**
(Simplify your expressions by cancelling when possible, but don't bother doing the division.
That is, leave your answers as fractions.)

(10) Find $f(3.1)$ by substituting $x = 3.1$ into the factored version of f. (Notice that this is significantly easier than what you did in question (4)!

(11) $f(3.01) =$

(12) $f(3.001) =$

(13) Guess $\lim\limits_{x \to 3^+} f(x) =$

(14) $f(2.9) =$

(15) $f(2.99) =$

(16) $f(2.999) =$

(17) Guess $\lim_{x \to 3^-} f(x) =$

(18) Guess $\lim_{x \to 3} f(x) =$

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Part 3: Limit as $x \to 2$

Using the factored form of f, compute the following values and guess the limits. **(No calculators)**
(Simplify your expressions by cancelling when possible.)

(19) $f(2.1) =$

(20) $f(2.01) =$

(21) $f(2.001) =$

(22) Describe in words the trend that you observe in (19), (20), (21).

(23) Does $\lim\limits_{x \to 2^+} f(x)$ exist? Explain.

(24) $f(1.9) =$

(25) $f(1.99) =$

(26) $f(1.999) =$

(27) Describe in words the trend that you observe in (24), (25), (26).

(28) Does $\lim_{x \to 2^-} f(x)$ exist? Explain.

(29) Does $\lim_{x \to 2} f(x)$ exist? Explain.

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