


MATH 2301 (Barsamian) GW07 Representations of Slopes

## Definition of Average Rate of Change

- words: the average rate of change of $f$ as the input changes from $a$ to $b$
- usage: $f$ is a function that is continuous on the interval $[a, b]$.
- meaning: the number $m=\frac{f(b)-f(a)}{b-a}$
- graphical interpretation: The number $m$ is the slope of the secant line that touches the graph of $f$ at the points $(a, f(a))$ and $(b, f(b))$.
- remark: The average rate of change $m$ is a number.


## Definition of Instantaneous Rate of Change

- words: the instantaneous rate of change of $f$ at $a$
- alternate words: the derivative of $f$ at $a$
- symbol: $f^{\prime}(a)$
- meaning: the number $m=\lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
- graphical interpretation: The number $m$ is the slope of the line tangent to the graph of $f$ at the point $(x, y)=(a, f(a))$.
- remark: The instantaneous rate of change $f^{\prime}(a)$ is a number.

Each expression in Column 2 represents a number $m$ that can be interpreted as the slope of a line on the graph of $f$. In each example, draw the line on the graph of $f$, or write the missing expression based on the line shown in the graph, and then give the value of the number $m$ represented by the expression.

| Example | Expression representing $m$ | Line whose slope is $m$ | Value of $m$ |
| :---: | :---: | :---: | :---: |
| (1) | the average rate of change of $f$ as the input changes from 1 to 5 |  | $m=$ |
| (2) | the derivative of $f$ at $x=1$ |  | $m=$ |


| Example | Expression representing $m$ | $\underline{\text { Line whose slope is } m}$ | Value of $m$ |
| :---: | :---: | :---: | :---: |
| (3) | the instantaneous rate of change of $f$ at $x=4$ |  | $m=$ |
| (4) | $\lim _{h \rightarrow 0} \frac{f(3+h)-f(3)}{h}$ |  | $m=$ |
| (5) | $\frac{f(4)-f(2)}{4-2}$ |  | $m=$ |
| (6) | $f^{\prime}(2)$ |  | $m=$ |
| (7) |  |  | $m=$ |

