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**MATH 2301 GW14: Comparing Two Solutions to an Absolute Extrema Problem**

Here is a problem about absolute extrema, along with two student solutions.

Find the absolute extrema of the function $f(x) = x^4 - 6x^2 + 5$ on the interval $[-2,3]$.

**Alan's Solution:**

| $x$ | $f(x)$ | |
|---|---|---|
| -2 | $f(-2) = (-2)^4 - 6(-2)^2 + 5 = \cdots = -3$ | MIN |
| -1 | $f(-1) = (-1)^4 - 6(-1)^2 + 5 = \cdots = 0$ | |
| 0 | $f(0) = (0)^4 - 6(0)^2 + 5 = \cdots = 5$ | |
| 1 | $f(1) = (1)^4 - 6(1)^2 + 5 = \cdots = 0$ | |
| 2 | $f(2) = (2)^4 - 6(2)^2 + 5 = \cdots = -3$ | MIN |
| 3 | $f(3) = (3)^4 - 6(3)^2 + 5 = \cdots = 32$ | MAX |

Absolute max of $y = 32$ (occurs at $x = 3$).

Absolute min of $y = -3$ (occurs at $x = -2$ and $x = 2$).

**Betty's Solution:**

Find Critical Numbers: $f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 4x(x + \sqrt{3})(x - \sqrt{3}) = 0$.

Critical numbers $x = 0, -\sqrt{3}, \sqrt{3}$

| Important $x$ values | $f(x)$ | |
|---|---|---|
| $-\sqrt{3}$ (critical) | $f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 5 = \cdots = -4$ | MIN |
| $x = 0$ (critical) | $f(0) = (0)^4 - 6(0)^2 + 5 = \cdots = 5$ | MAX |
| $x = -\sqrt{3}$ (critical) | $f(\sqrt{3}) = (\sqrt{3})^4 - 6(\sqrt{3})^2 + 5 = \cdots = -4$ | MIN |

Absolute max of $y = 5$ (occurs at $x = 0$).

Absolute min of $y = -4$ (occurs at $x = -\sqrt{3}$ and $x = \sqrt{3}$).

**(Note: There are no arithmetic errors in either student's work.)**

**Comment on the two solutions. What is good and bad about each? Write on back.**