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MATH 2301 GW21: Newton's Method

Key Idea 4.1.2 Newton's Method

Given: A function f that is differentiable on an interval I and that has a root in I. That is, it is known that there exists an $x = r$ somewhere in I such that $f(r) = 0$.

Goal: Find an approximate value for the root r, accurate to d decimal places.

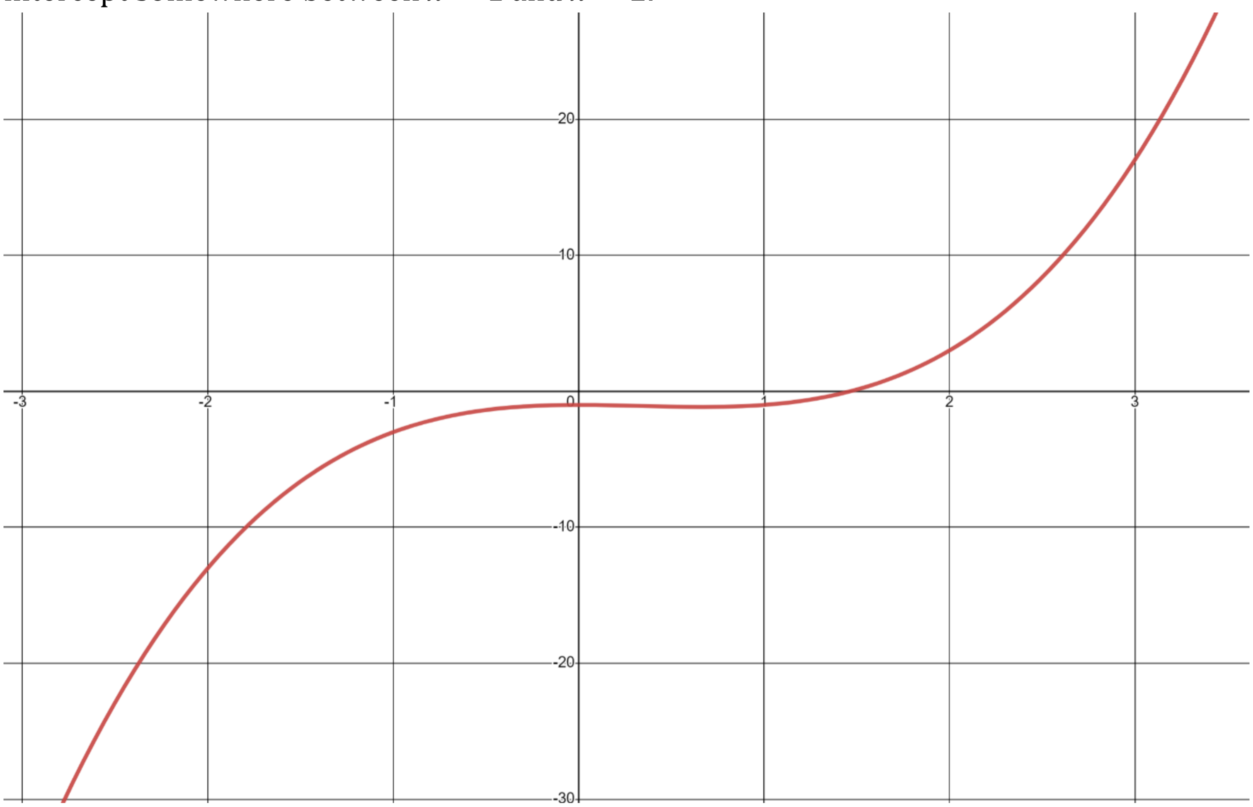
Step 1: Choose a value x_0 as an initial approximation of the root. (This is often done by looking at a graph.)

Step 2: Create successive approximations iteratively; given an approximation x_n, compute the next approximation x_{n+1} by using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Step 3: Stop the iterations when successive approximations do not differ in the first d places after the decimal point.

Let $f(x) = x^3 - x^2 - 1$. Observe that the graph of $f(x)$ obtained from Desmos shows an x intercept somewhere between $x = 1$ and $x = 2$.



The goal is to use Newton's method to find a root of f. That is, the goal is to find an $x = r$ such that $f(r) = 0$. You will do the first two iterations only, using the initial approximation $x_0 = 1$. That is, you will find x_1 and x_2.

The Group Work continues on back ➔

For the function $f(x) = x^3 - x^2 - 1$,

(a) Compute $f'(x)$

(b) Fill out the following table. (Do the details below.)

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
0	$x_0 = 1$			$x_1 =$
1	$x_1 =$			$x_2 =$
2	$x_2 =$			

The Group Work continues on next page ➔

(C) A zoomed-in graph of $f(x)$ is shown below. You'll illustrate your results on this graph.

- Put a point at $(x_0, 0)$
- Put a point at $(x_0, f(x_0))$
- Put a point at $(x_1, 0)$.
- Draw the line that passes through $(x_0, f(x_0))$ and $(x_1, 0)$. This line should appear to be tangent to the graph of $f(x)$ at the point $(x_0, f(x_0))$.
- Put a point at $(x_1, f(x_1))$
- Put a point at $(x_2, 0)$.
- Draw the line that passes through $(x_1, f(x_1))$ and $(x_2, 0)$. This line should appear to be tangent to the graph of $f(x)$ at the point $(x_1, f(x_1))$.