													,											
													,											
													,											
L	A	S	T		N	A	M	E	S				,	F	I	R	S	T		N	A	M	E	S

MATH 2301 GW28: Computing Definite Integrals By Using Geometry

Symbol: $\displaystyle\int_{x=a}^{x=b} f(x)dx$

Spoken: The definite integral of $f(x)$ from $x = a$ to $x = b$.

Informal Definition: the *signed area* of the region between the graph of $f(x)$ and the x-axis, from $x = a$ to $x = b$.

Remark: This is an *informal* definition because we have only have a notion of area for certain basic geometric shapes. For now, this definition of defnite integral can only be used in situations where the region between the graph of $f(x)$ and the x-axis, from $x = a$ to $x = b$, is made up of basic geometric shapes. In those situations, the value of the definite integral can be found by using familiar geometric formulas to compute the areas of the shapes that make up the region.

Sometimes, the integrand $f(x)$ is given by a *formula* whose graph is made up basic geometric shapes. In these situations, the value of the integral can be found by first *graphing* $f(x)$, then using geometric area formulas to find the unsigned areas of the shapes that make up the region between the graph of $f(x)$ and the x axis.

Starting on the next page, you will do three integrals of this type

[1] For the integral $\displaystyle\int_{x=2}^{x=8} \left(\frac{1}{2}\right)x + 3dx$

(a) Graph the integrand.

(b) Shade the region between the graph of $f(x)$ and the x axis that corresponds to the integral.

(c) Use geometric formulas to find areas of the shaded shapes. Then find the value of the integral.

[2] For the integral $\displaystyle\int_{x=2}^{x=8} \left(\frac{1}{2}\right)x - 2dx$

(a) Graph the integrand.

(b) Shade the region between the graph of $f(x)$ and the x axis that corresponds to the integral. (Shade the regions above the x axis one color and the regions below the axis a different color.)

(c) Use geometric formulas to find areas of the shaded shapes. Then find the value of the integral.

[3] For the integral $\int_{x=1}^{x=11} \sqrt{25-(x-6)^2}\,dx$

(a) Graph the integrand.

(b) Shade the region between the graph of $f(x)$ and the x axis that corresponds to the integral. (Shade the regions above the x axis one color and the regions below the axis a different color.)

(c) Use geometric formulas to find areas of the shaded shapes. Then find the value of the integral.

Hint:

- The equation $(x-a)^2+(y-b)^2=r^2$ describes a circle centered at (a,b) with radius r.
- The equation $(x-a)^2+y^2=r^2$ describes a circle centered on the x axis at $(a,0)$ with radius r.
- The equation $y=\sqrt{r^2-(x-a)^2}$ describes the upper semicircle of a circle centered on the x axis at $(a,0)$ with radius r.

.