Reference 1: Definition of Limit

The Definition of *Limit*

Symbol: $\lim_{x \to c} f(x) = L$.

Spoken: "The limit, as x approaches c, of $f(x)$ is L."

Less-Abbreviated Symbol: $f(x) \to L$ as $x \to c$.

Spoken: "$f(x)$ approaches L as x approaches c."

Usage: x is a variable, f is a function, c is a real number, and L is a real number.

Informal Meaning: as x gets closer and closer to c, but not equal to c, the value of $f(x)$ gets closer and closer to L (and may actually equal L).

Precise Meaning: For every number $\epsilon > 0$, there exists a number $\delta > 0$ such that

$$\text{If } x \neq c \text{ and } |x - c| < \delta, \text{ then } |f(x)| < \epsilon$$

Graphical Significance: The graph of f appears to be heading for location $(x, y) = (c, L)$ from both sides.

The Definition of *Limit from the Left*

Symbol: $\lim_{x \to c^-} f(x) = L$.

Spoken: "The limit, as x approaches c from the left, of $f(x)$ is L."

Less-Abbreviated Symbol: $f(x) \to L$ as $x \to c^-$.

Spoken: "$f(x)$ approaches L as x approaches c from the left."

Usage: x is a variable, f is a function, c is a real number, and L is a real number.

Informal Meaning: as x gets closer and closer to c, but less than c, the value of $f(x)$ gets closer and closer to L (and may actually equal L).

Precise Meaning: For every number $\epsilon > 0$, there exists a number $\delta > 0$ such that

$$\text{If } x < c \text{ and } |x - c| < \delta, \text{ then } |f(x)| < \epsilon$$

Graphical Significance: The graph of f appears to be heading for location $(x, y) = (c, L)$ from the left.

There is an analogous Definition of *Limit from the Right*.