Reference 2: Using The Squeeze Theorem

The Squeeze Theorem (as stated in the book)

If f, g, h are functions on an open interval I containing c such that

$f(x) \leq g(x) \leq h(x)$ for all x in I (except possibly at $x = c$)

and such that

$$\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x),$$

Then $\lim_{x \to c} g(x) = L$.

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The Squeeze Theorem (presented another way)

If we know that all this stuff is true (these are the "hypotheses"):

- There is some real number c.
- There is an interval I containing c.
- There is a function $f(x)$.
- There is a function $g(x)$.
- There is a function $h(x)$.
- The functions f, g, h satisfy $f(x) \leq g(x) \leq h(x)$ for all x in I (except possibly at $x = c$)
- The function f has limit $\lim_{x \to c} f(x) = L$.
- The function h has the same limit $\lim_{x \to c} h(x) = L$.

Then we are allowed to say these words (this is the "conclusion"):

- "The function g has limit $\lim_{x \to c} g(x) = L$."

When we “use” this theorem (or any other), we start by verifying explicitly that all of the hypotheses are satisfied in our specific situation. If they are satisfied, then we write the conclusion, adapted to our specific situation.

Generic Hypotheses	Our Specific Hypotheses
the real number c	
the interval I containing c	
the function $f(x)$	
the function $g(x)$.	
the function $h(x)$.	
verification that functions f, g, h satisfy $f(x) \leq g(x) \leq h(x)$ for all x in I (except possibly at $x = c$)	
Function f has limit $\lim_{x \to c} f(x) = L$.	
Function h has limit $\lim_{x \to c} h(x) = L$.	
Generic Conclusion	**Our Specific Conclusion**
Function $g(x)$ has limit $\lim_{x \to c} g(x) = L$	$Function$ _____$(x) =$ ________________ $has\ limit$ $\lim_{x \to \_\_\_}$ ____$(x) =$ ______ .

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