

Reference 3: Using The Intermediate Value Theorem

The Theorem as stated in many books. (Similar, but not identical, to the statement in our book):

The Intermediate Value Theorem

Suppose that f is continuous on the closed interval $[a, b]$ and such that where $f(a) \neq f(b)$.
Then for any real number y between $f(a)$ and $f(b)$,
there exists some number c in (a, b) such that $f(c) = y$.

Presented another way:

The Intermediate Value Theorem

If we know that all this stuff is true (these are the “hypotheses”):

- There is a closed interval $[a, b]$.
- The function f is continuous on the closed interval $[a, b]$.
- The numbers $f(a) \neq f(b)$.
- The number y is a real number between $f(a)$ and $f(b)$.

Then we are allowed to say these words (this is the “conclusion”):

- “There exists at least one number c in (a, b) such that $f(c) = y$.”

When we “use” this theorem (or any other), we start by verifying explicitly that all of the hypotheses are satisfied in our specific situation. If they are satisfied, then we write the conclusion, adapted to our specific situation.

Generic Hypotheses	Our Specific Hypotheses
the real number “ a ”	
real number “ b ” such that $a < b$	
the function $f(x)$	
verification that f is continuous on the closed interval $[a, b]$.	
the value of $f(a)$	
the value of $f(b)$	
confirmation that $f(a) \neq f(b)$	
the real number “ y ”	
verification that y is between $f(a)$ and $f(b)$	
Generic Conclusion	Our Specific Conclusion
“There exists at least one number c in the open interval (a, b) such that $f(c) = y$.”	“There exists at least one number c in the open interval _____ such that $f(c) = \underline{\hspace{1cm}}$.”