Reference R05: Graphing Strategy

General Idea:

- Start with simplest, least sophisticated analysis, then proceed to more sophisticated.
- If a step is not easy (for instance, if it is not clear how to factor $f(x)$), then skip that step.

Step 1. Analyze $f(x)$.

- Find the y-intercept
- Find the x-intercepts.
- Determine the end-behavior (Are there horizontal asymptotes? Or do the ends of the graph go up or down?) by finding $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Determine location of any vertical asymptotes
- Make a sign chart for f and use it to determine where f is positive, negative, or zero.

Step 2. Analyze $f'(x)$.

- Find $f'(x)$, factor it, and then find the partition numbers for $f'(x)$.
- Construct a sign chart for $f'(x)$ and use it to determine the x coordinates where graph of f has a horizontal tangent line, the intervals on which f is increasing and decreasing, and the x coordinates of all relative maxima and minima.
- Find the y coordinates of all relative maxima and minima.

Step 3. Analyze $f''(x)$.

- Find $f''(x)$, factor it, and then find the partition numbers for $f''(x)$.
- Construct a sign chart for $f''(x)$ and use it to determine the intervals on which f is concave up and concave down, and to find the x coordinates of all inflection points.
- Find the y coordinates of all inflection points.

Step 4: Sketch the graph of f.

- Draw any asymptotes as dotted lines, and label them with their line equations.
- Plot the axis intercepts, relative maxima and minima, and inflection points, and label them with their (x, y) coordinates.
- Using the other information from steps 1, 2, and 3, draw the graph.